Modeling Immigration with a Discrete Dynamical System

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- How the government uses money to decrease the number of undocumented workers

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- Any person who was just born, legalized, or has moved into the country does not contribute any money to the government in that same time interval.
- The only money the government spends is money on removing undocumented workers.

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$$G_n = G_{n-1} + m_L(1 - d_L)L_{t-1} + m_U(1 - d_U)U_n - X_{n-1}$$

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$$X_n = kU_n$$

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- The number of legal residents who enter the country is constant every time interval.
- A child of an undocumented worker is a legal resident; that child cannot have children, die, or leave the country in the same time interval.

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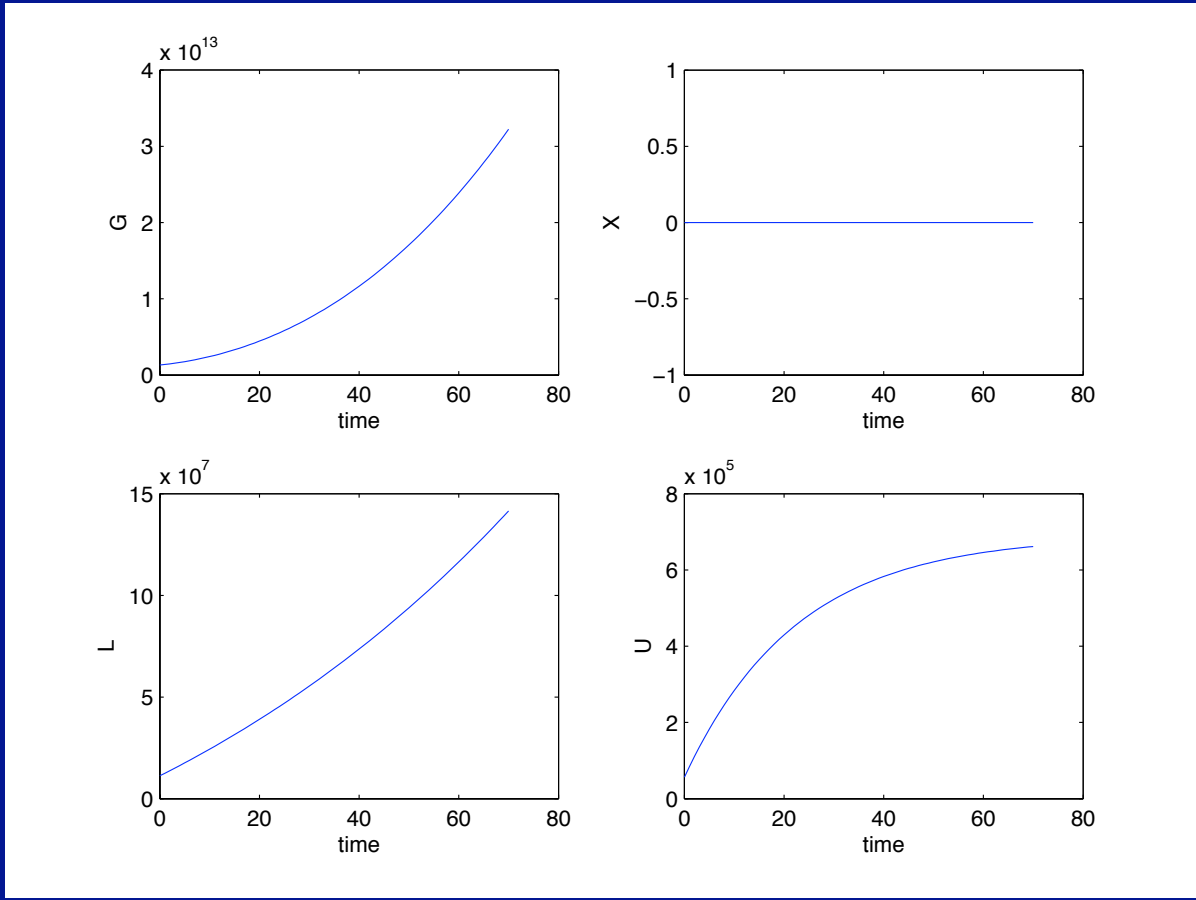
- We look at plots for different values of k.
- $e = 10^{-4}$
- $G_0 = 1.308608 * 10^{12}$
- $L_0 = 1.1271743 * 10^{7}$
- $U_0 = 5.6488 * 10^{4}$

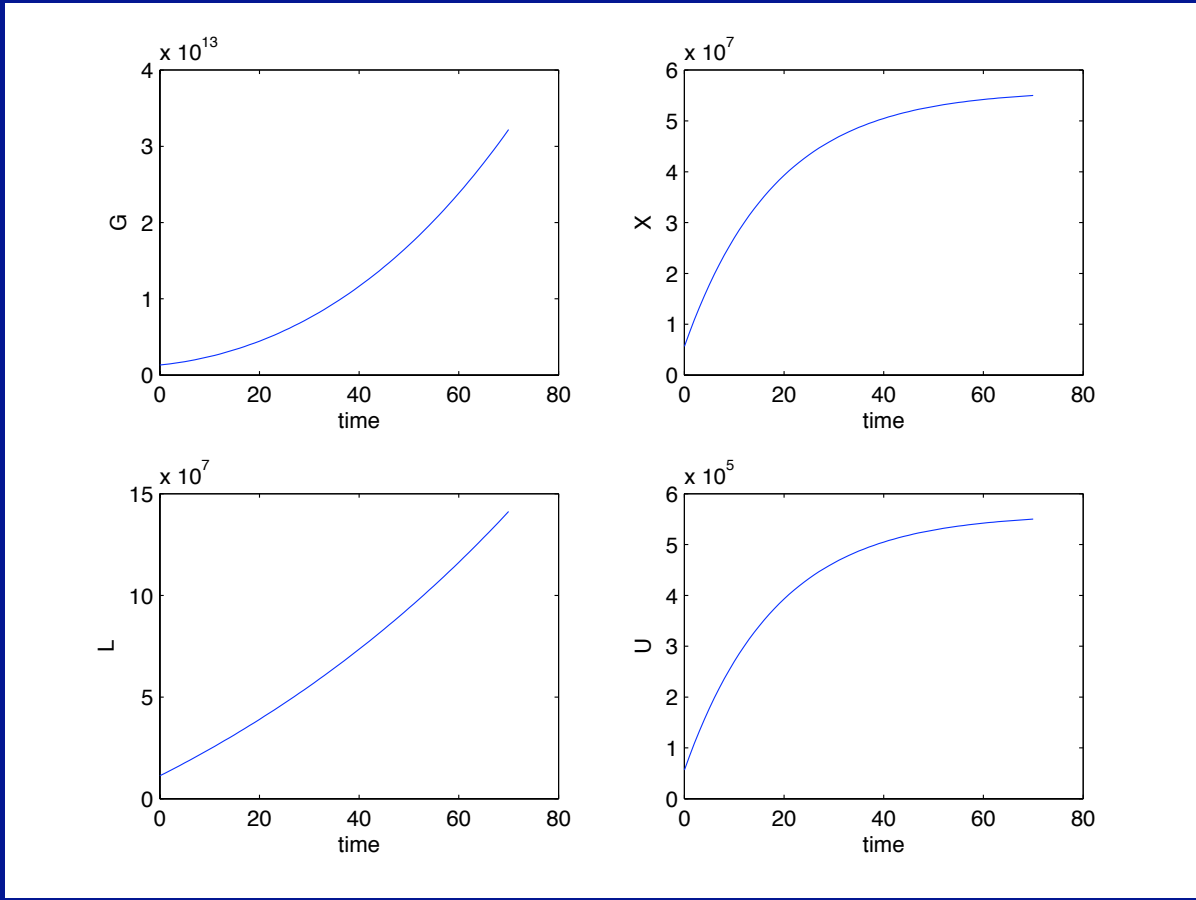
Different values of k

When $k = 0$



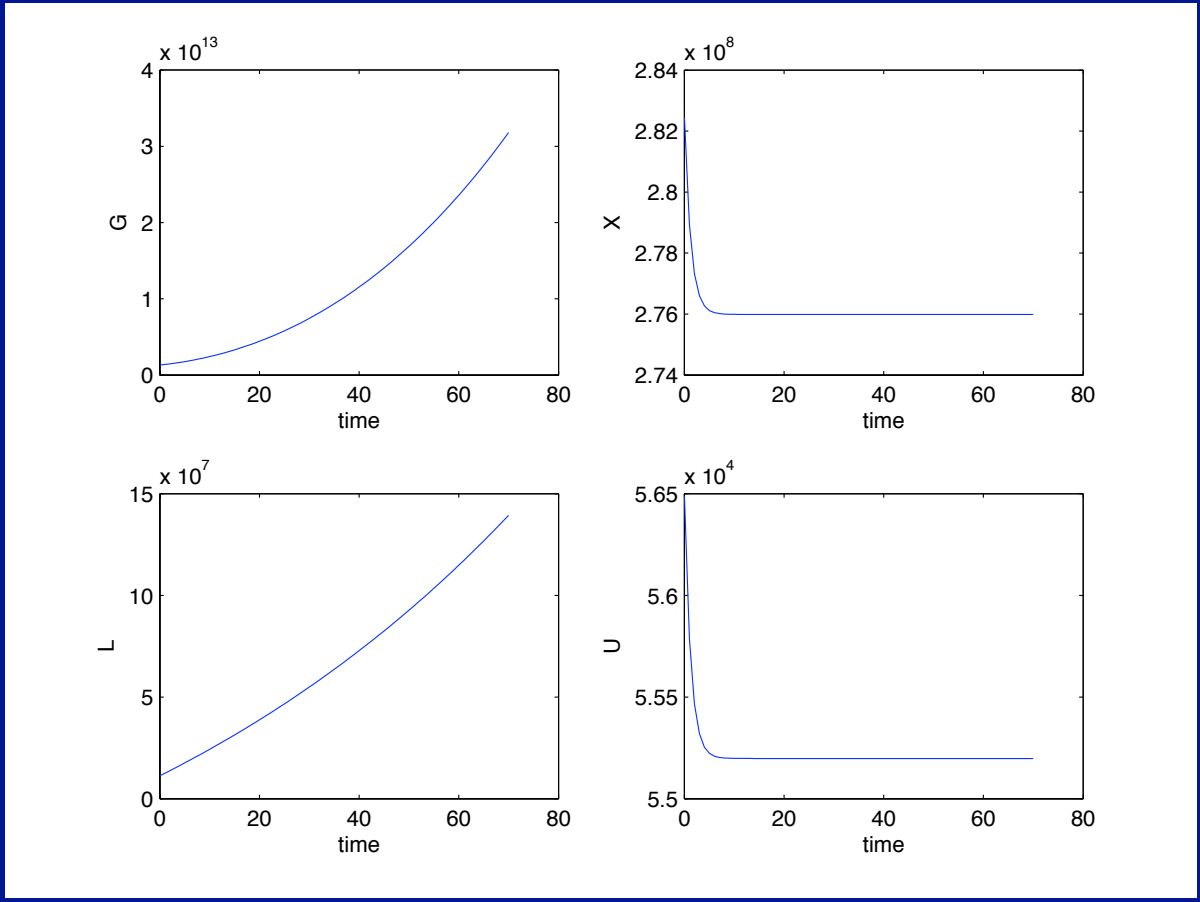
Different values of k

When $k = 100$



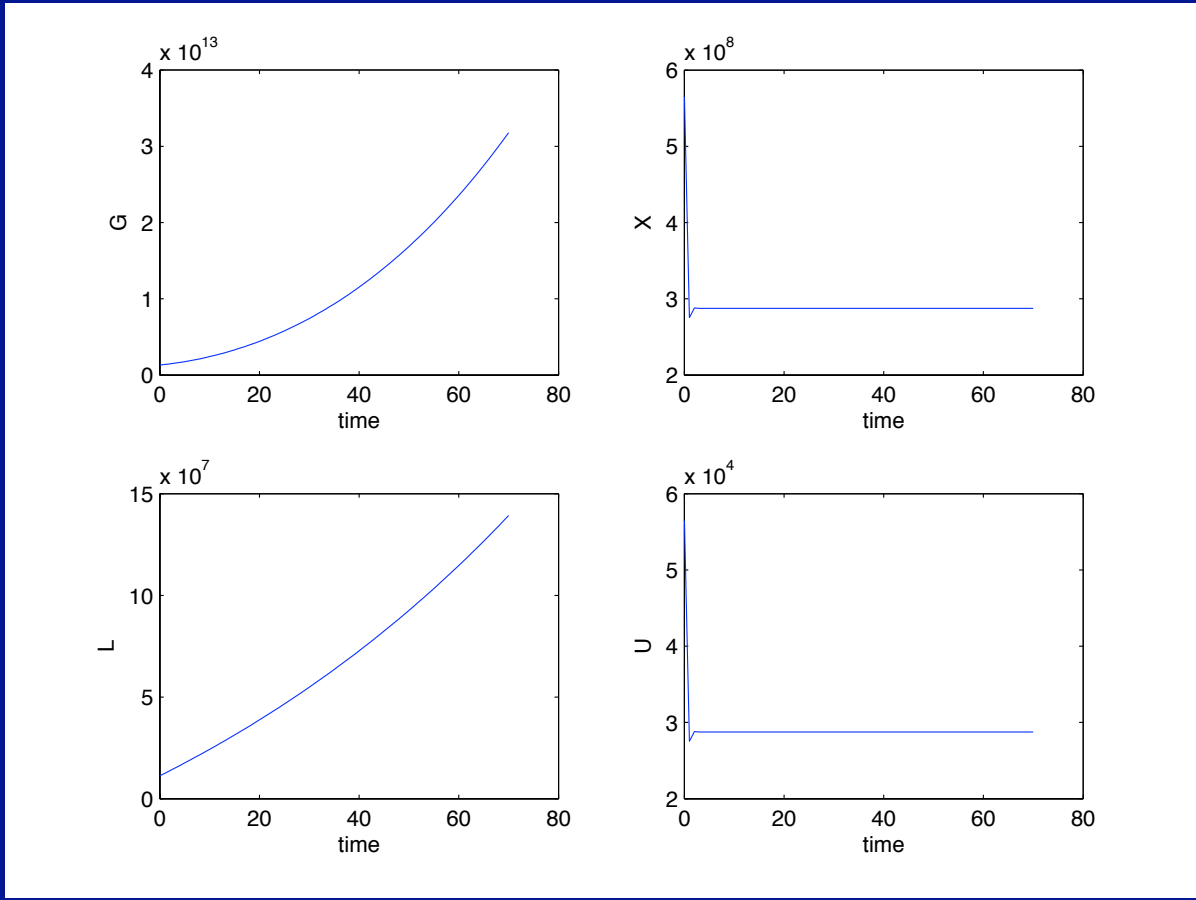
Different values of k

When $k = 5000$



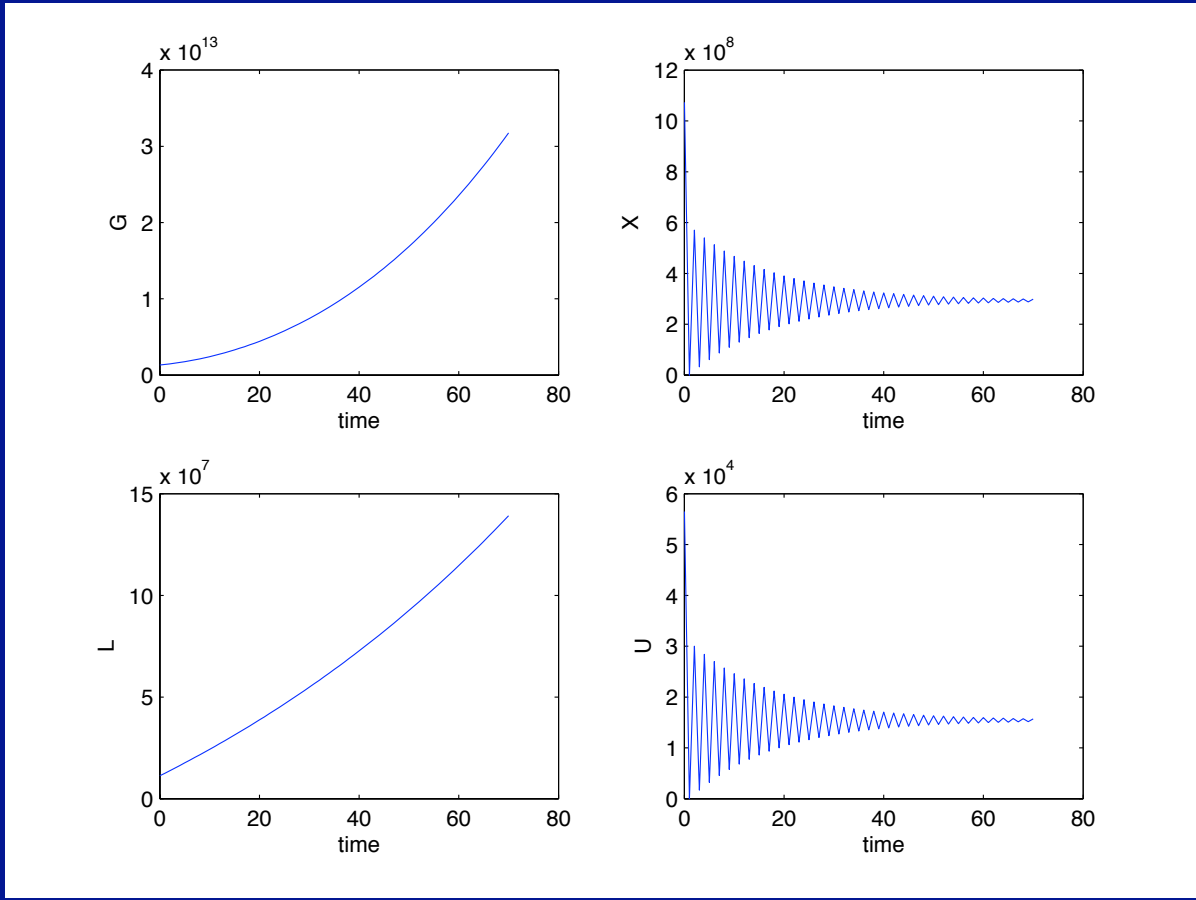
Different values of k

When $k = 10000$



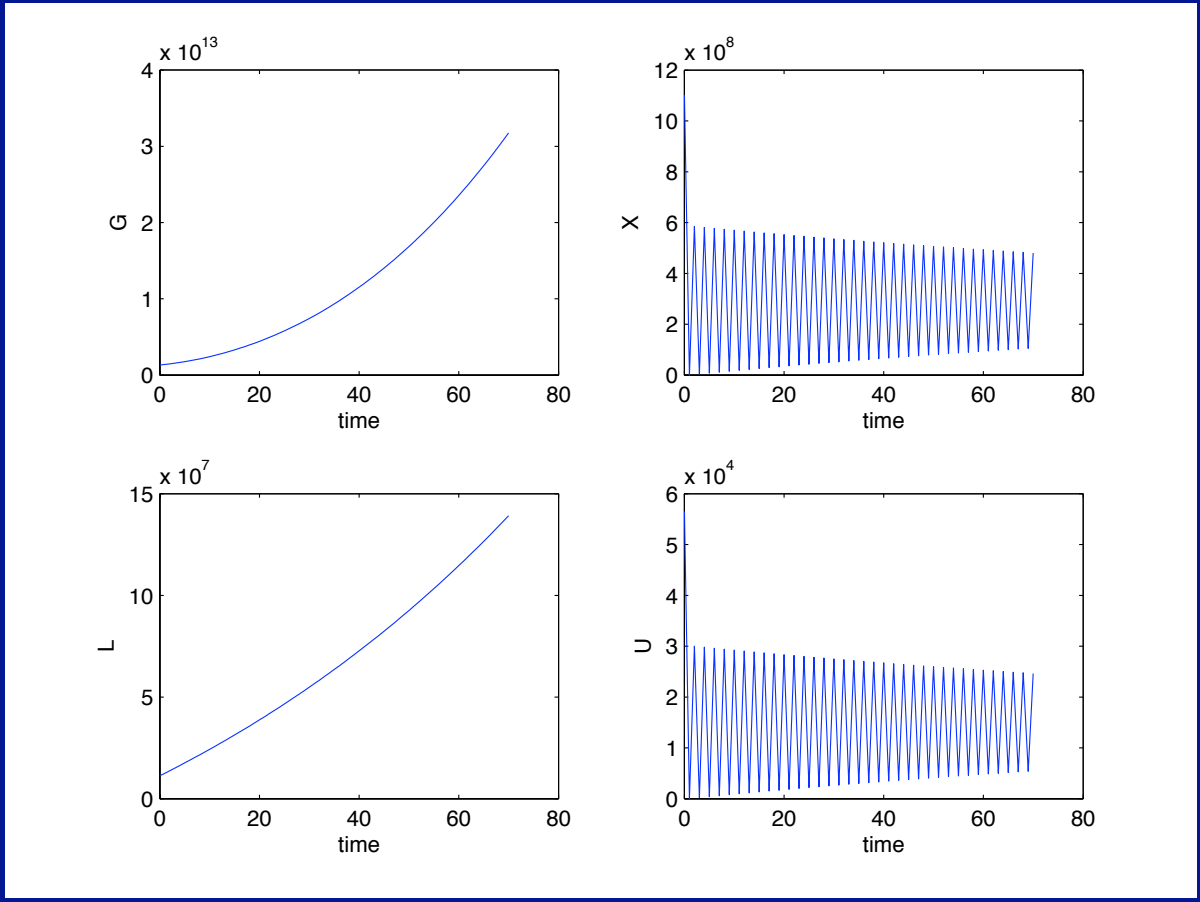
Different values of k

When $k = 19000$



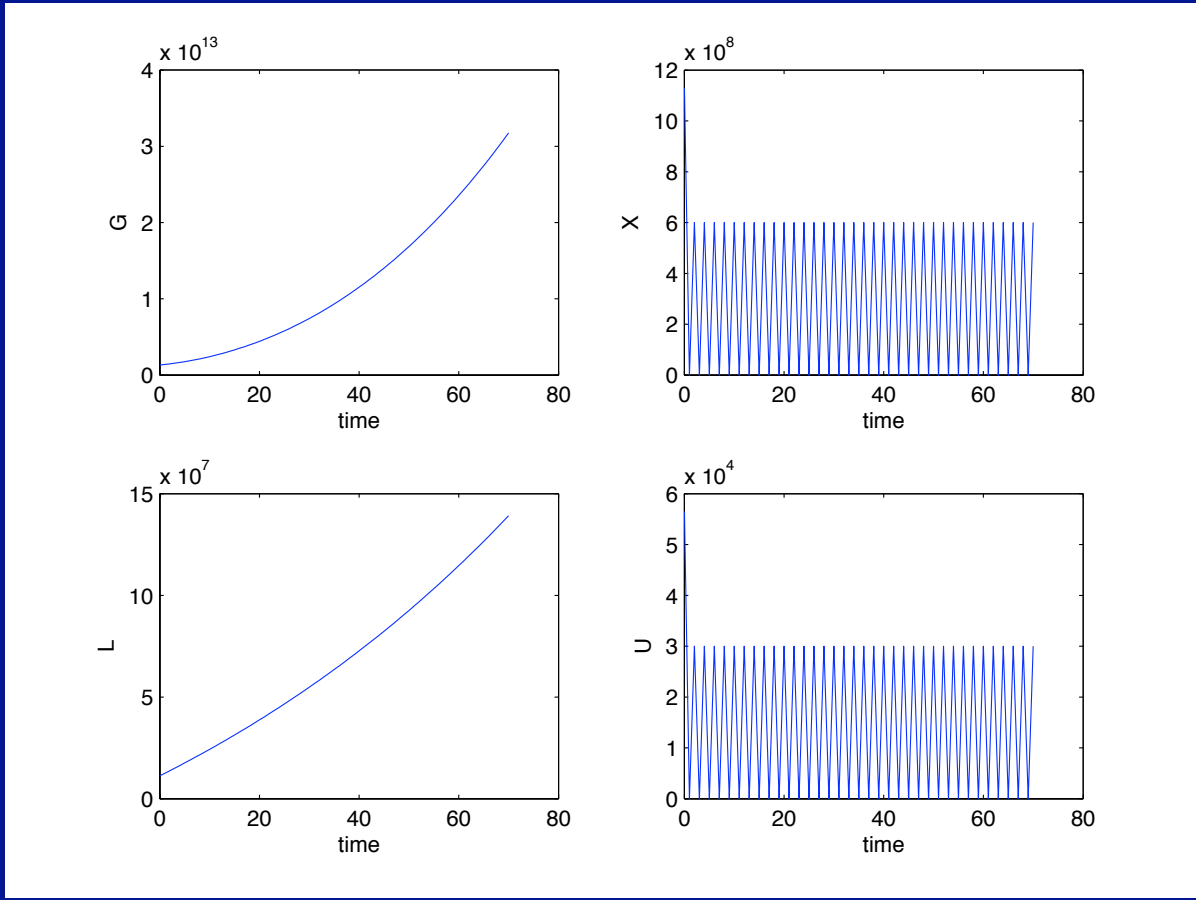
Different values of k

When $k = 19500$



Different values of k

When $k = 20000$

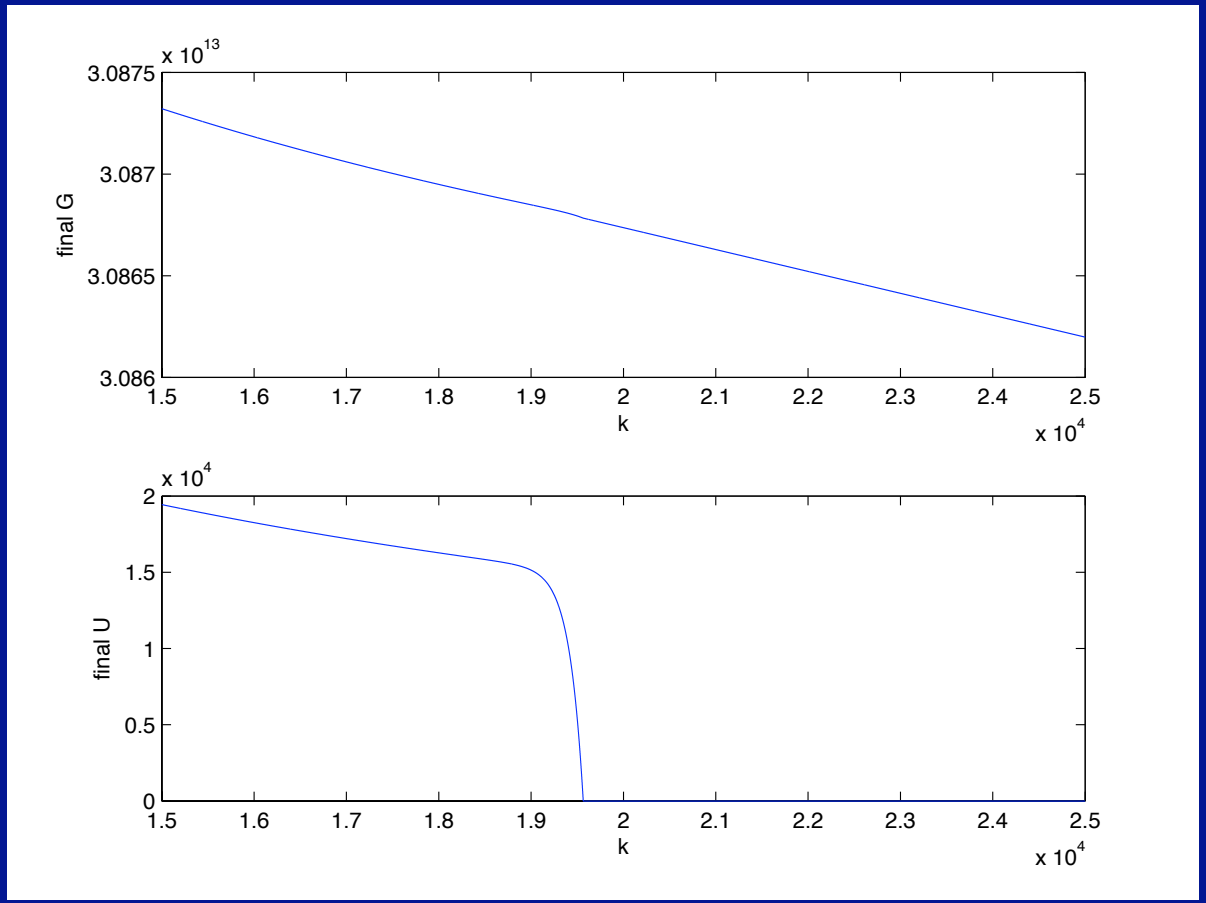


Different Values of k

Let's plot G_{final} and U_{final} for different values of k. But U oscillates for big k! Hence, let's plot $G_{final-1}$ and $U_{final-1}$ also.

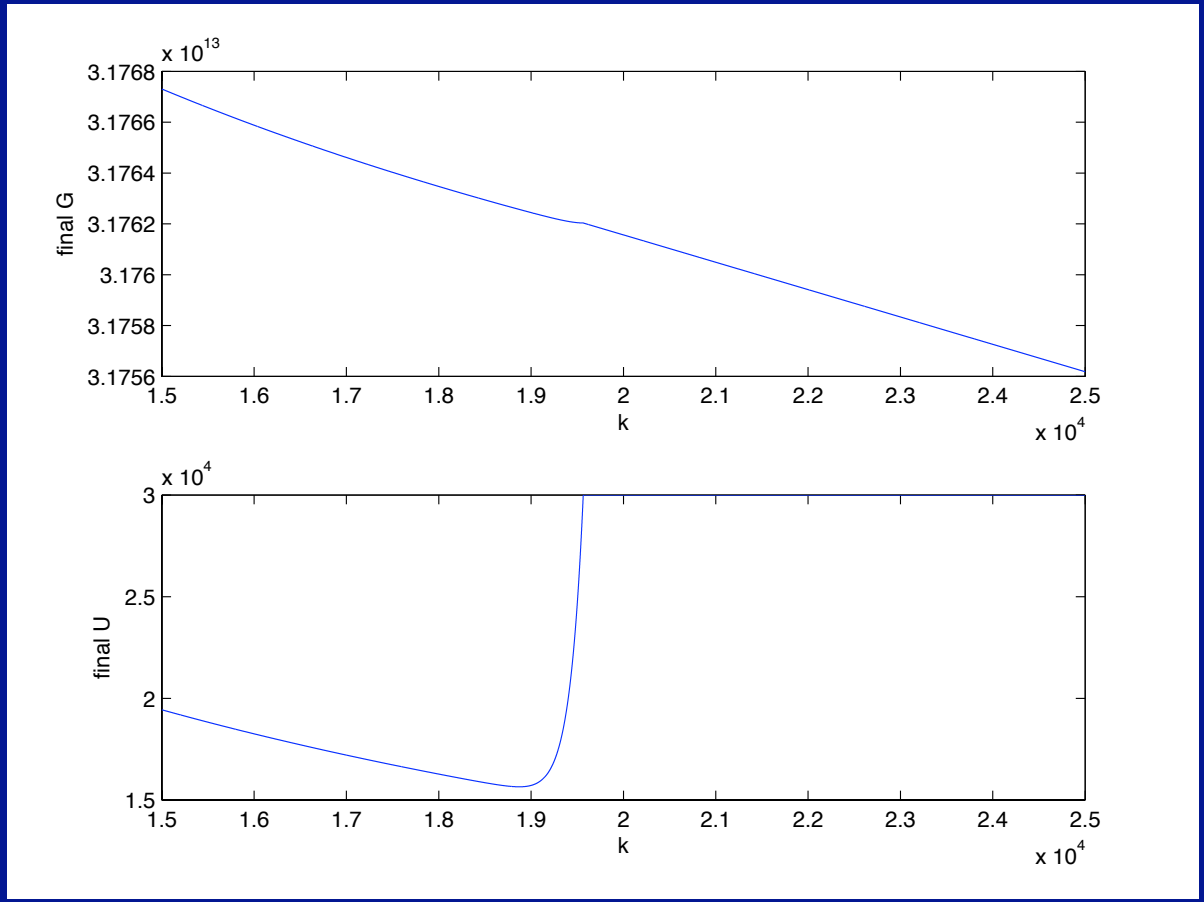
Different Values of k

For G_{final} and U_{final}...



Different Values of k

For $G_{final-1}$ and $U_{final-1}$...



Ways to Improve

- Deportation rate should not be linear with amount spent.
- Legal immigration rate should not constant.

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Another Model to Explore:

$$L_n = (b_L - d_L - l_L)L_{n-1} + f(G_{n-1})N_L + (b_U + g_U)U_{n-1}$$

$$U_n = U_{n-1} - (g_U + d_U)U_{n-1} + N_U - eh(X_{n-1})$$

References

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- US Census Website http://www.census.gov/population/socdemo/foreign/p20-534/tab0101.pdf
- Yong, Darryl. Harvey Mudd College