Indian Rope Trick

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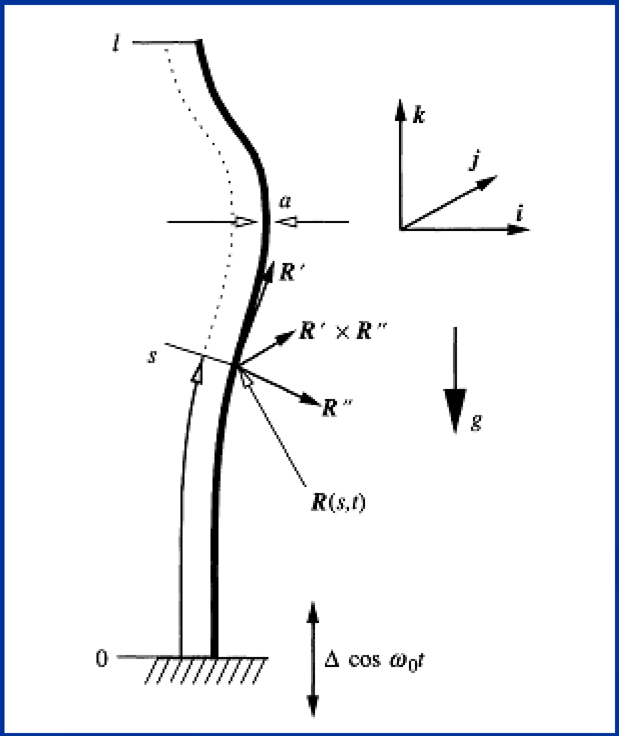
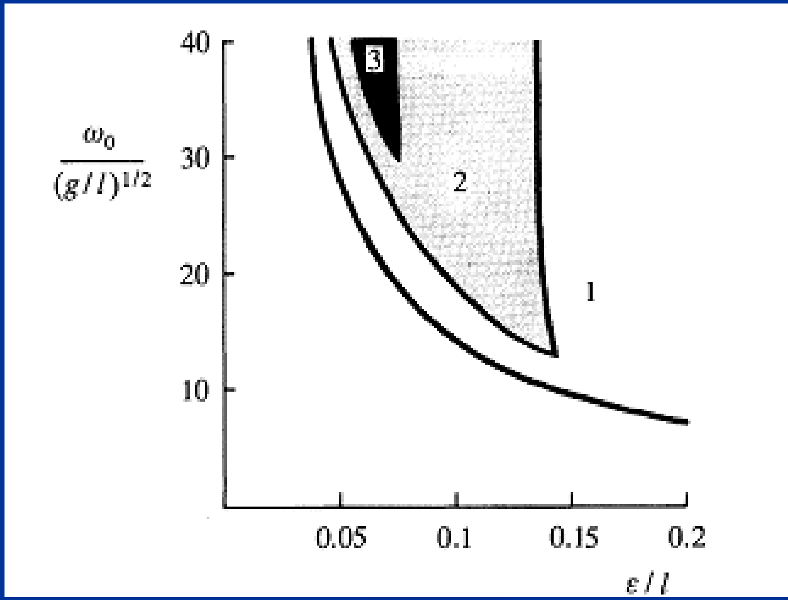
Inverted String

- Experimentally found that similar to the inverted pendulum, an inverted string also stands on end.



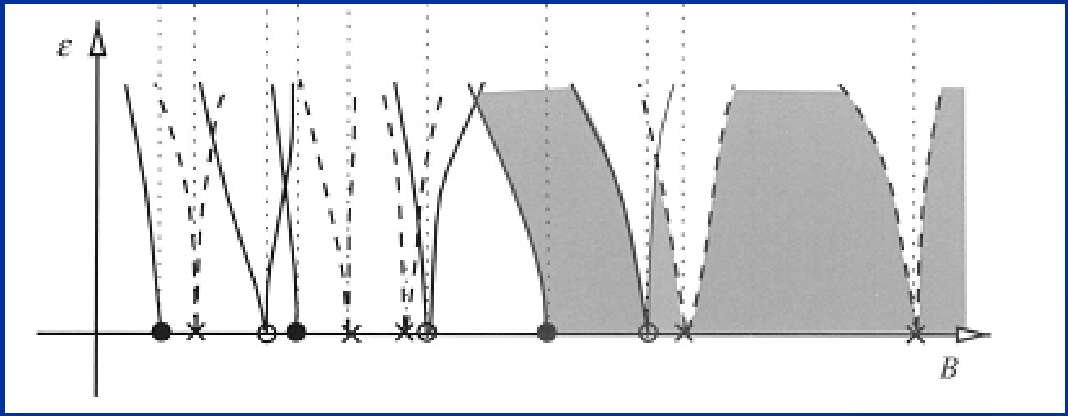
String Problems

- Mathematically found that a series of pendulums can also be stabilized with an oscillating pivot point.
- However, the required frequency goes to infinity for a continuous string.



Solution: Stiffness

- It was recently shown that by adding a "stiffness" term to the string PDE allowed stable solutions to be solved.



Equations of Motion

- These are the equations of motion:

$$\bar{D}^2\boldsymbol{r} - \boldsymbol{k}\varepsilon\cos\bar{t} = \delta[(\bar{T}\boldsymbol{r}')' + \bar{\boldsymbol{V}}' - \boldsymbol{k}],$$
$$\tfrac{1}{4}(a/\ell)^2\boldsymbol{r}' \times \bar{D}^2\boldsymbol{r}' = \delta(\bar{\boldsymbol{M}}' + \boldsymbol{r}' \times \bar{\boldsymbol{V}}),$$
$$\bar{\boldsymbol{M}} = \bar{B}(\boldsymbol{r}' \times \boldsymbol{r}''), \quad \boldsymbol{r}' \cdot \boldsymbol{r}' = 1, \quad \boldsymbol{r}' \cdot \bar{\boldsymbol{V}} = 0,$$

$$\bar{\boldsymbol{V}} = \delta B[\boldsymbol{r}' \times (\boldsymbol{r}' \times \boldsymbol{r}'''')].$$

Where

$$\left.\begin{array}{c} \bar{s} = s/\ell, \quad \bar{\boldsymbol{R}} = \boldsymbol{R}/\ell, \quad \bar{t} = \omega_0 t, \\ \bar{T} = \dfrac{T}{mg\ell}, \quad \bar{\boldsymbol{V}} = \dfrac{\boldsymbol{V}}{mg\ell}, \quad \bar{B} = \dfrac{B}{mg\ell^3}, \quad \delta = \dfrac{g}{\omega_0^2\ell}, \quad \bar{D} = \dfrac{D}{\omega_0}. \end{array}\right\}$$

$$y_{tt} = c^2 y_{xx} - \kappa y_{xxxx} + \frac{g}{\sqrt{2}} y$$

Finite Difference Method?

- Attempted to use a Finite Difference Method, but couldn't implement the nonlinear sine/cosine terms
- Tried to linearize equations, but realized that the oscillation at the base would be lost.

$$y_{tt} = c^2 y_{xx} - \kappa y_{xxxx} + \frac{g}{\sqrt{2}} y$$

Plans

- Use the Spectral Difference Method
- Hope to find convergence of stability towards a specific frequency