Insect Swarms and Chemoattractants

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Discrete Model

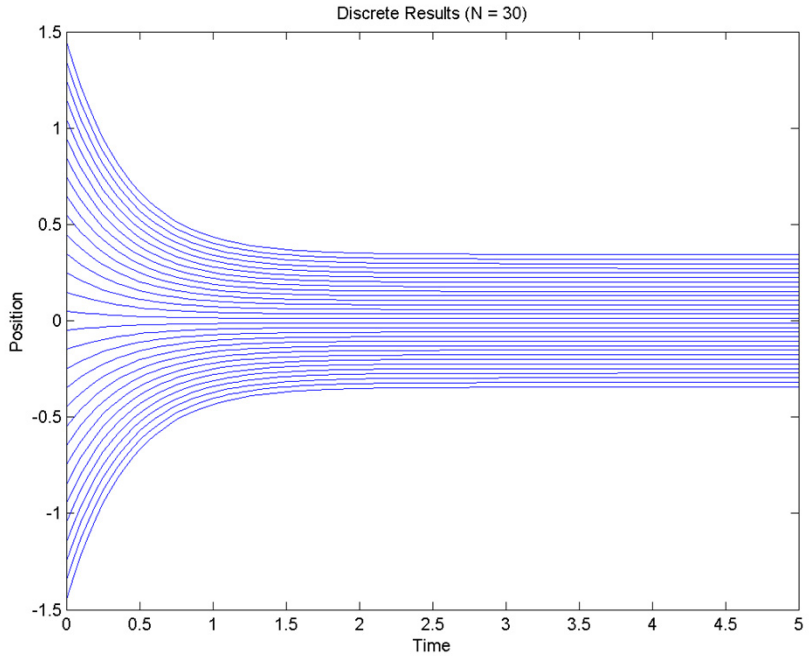
- System of *N* bugs in a 1-D domain
- Governing equation:

$$\frac{dx_i}{dt} = F_C(x_i) + \sum_{j \neq i} m_j F_S(x_i - x_j)$$

- Variables:
  - x_i = position of ith bug
  - m_i = “social mass” of ith bug
  - F_C = chemoattraction force
  - F_S = social repulsion force

Discrete Model: Numerical Approach

- Simple ODE solver
  - Uses Matlab's ode45 routine



Continuous Model

- Now consider a continuous swarm density $\rho(x,t)$
- Governing equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(v\rho) = 0$$

$$\text{where} \quad v(x,t) = F_C(x) + \int_{-\infty}^{\infty} \rho(z,t) F_S(x-z)\, dz$$

- Expectation: behavior similar to discrete model

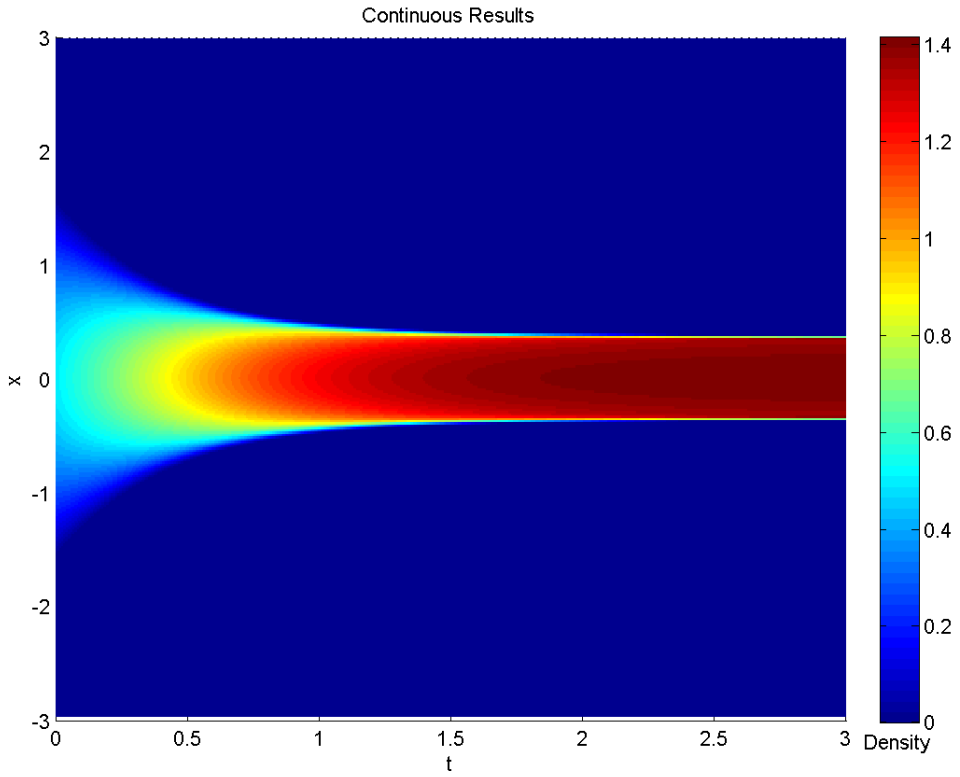
Continuous Model: Numerical Approach

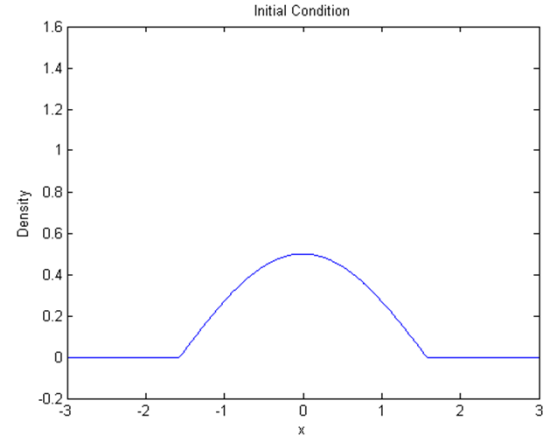
- With finite differences, PDE becomes:

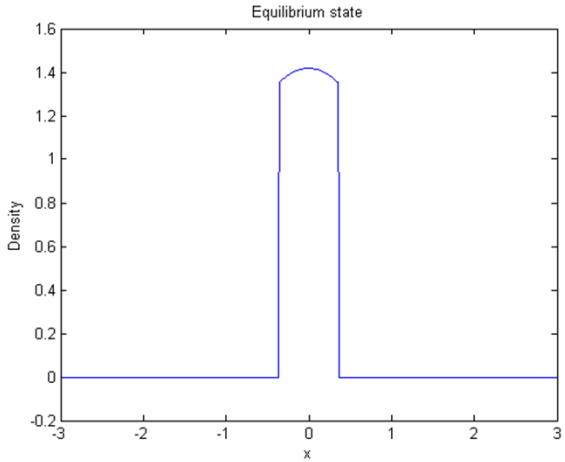
$$\frac{\rho_i^{j+1} - \rho_i^j}{\Delta t} + \frac{(v\rho)_{i+1}^j - (v\rho)_{i-1}^j}{2\Delta x} = 0$$

- Uses trapezoid rule to compute integral expression in $v(x,t)$
- Need to enforce positivity and conservation of mass
- Revised version uses 4th-order Runge-Kutta for time-stepping

Continuous Model: Results

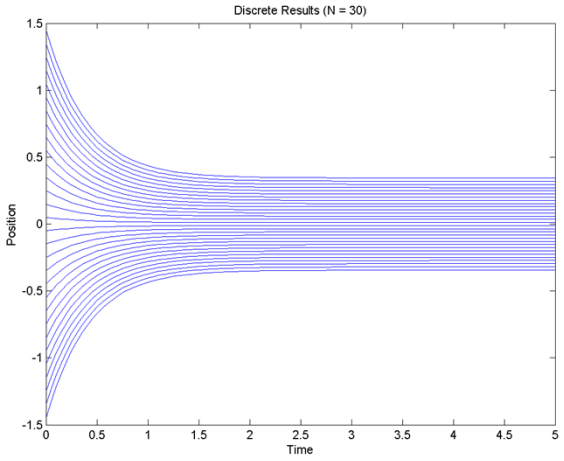


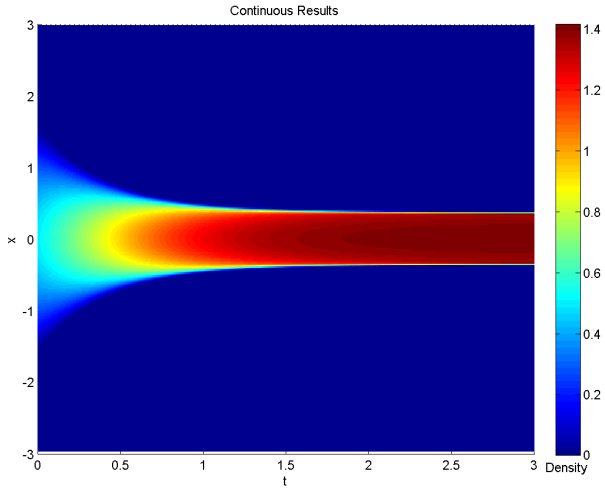




Comparing the Two Models

- The results from the two models seem to agree...





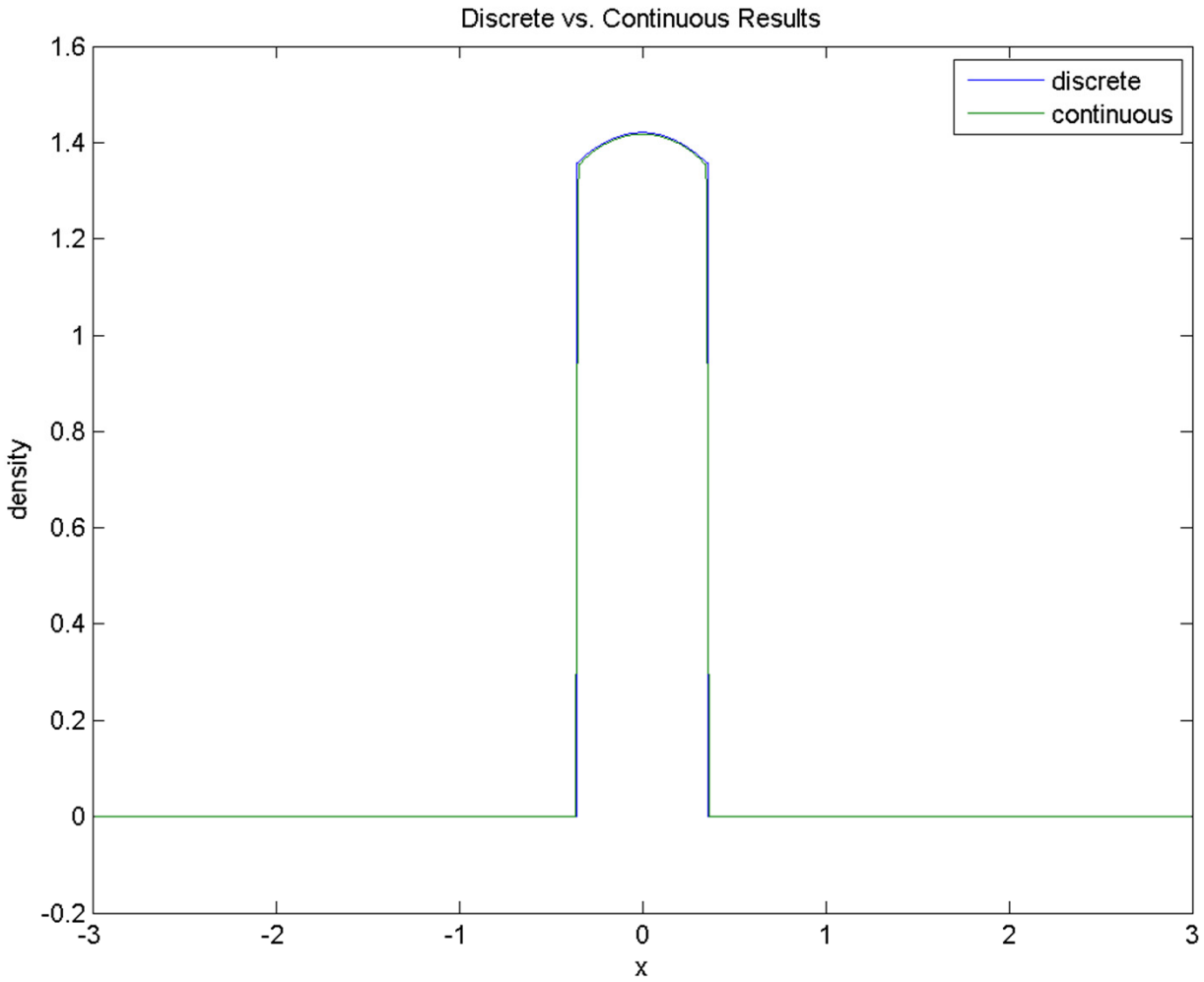
Comparing the Two Models

- But what about quantitative comparison?
- Density not well-defined in discrete case
- Idea – Use cumulative mass:

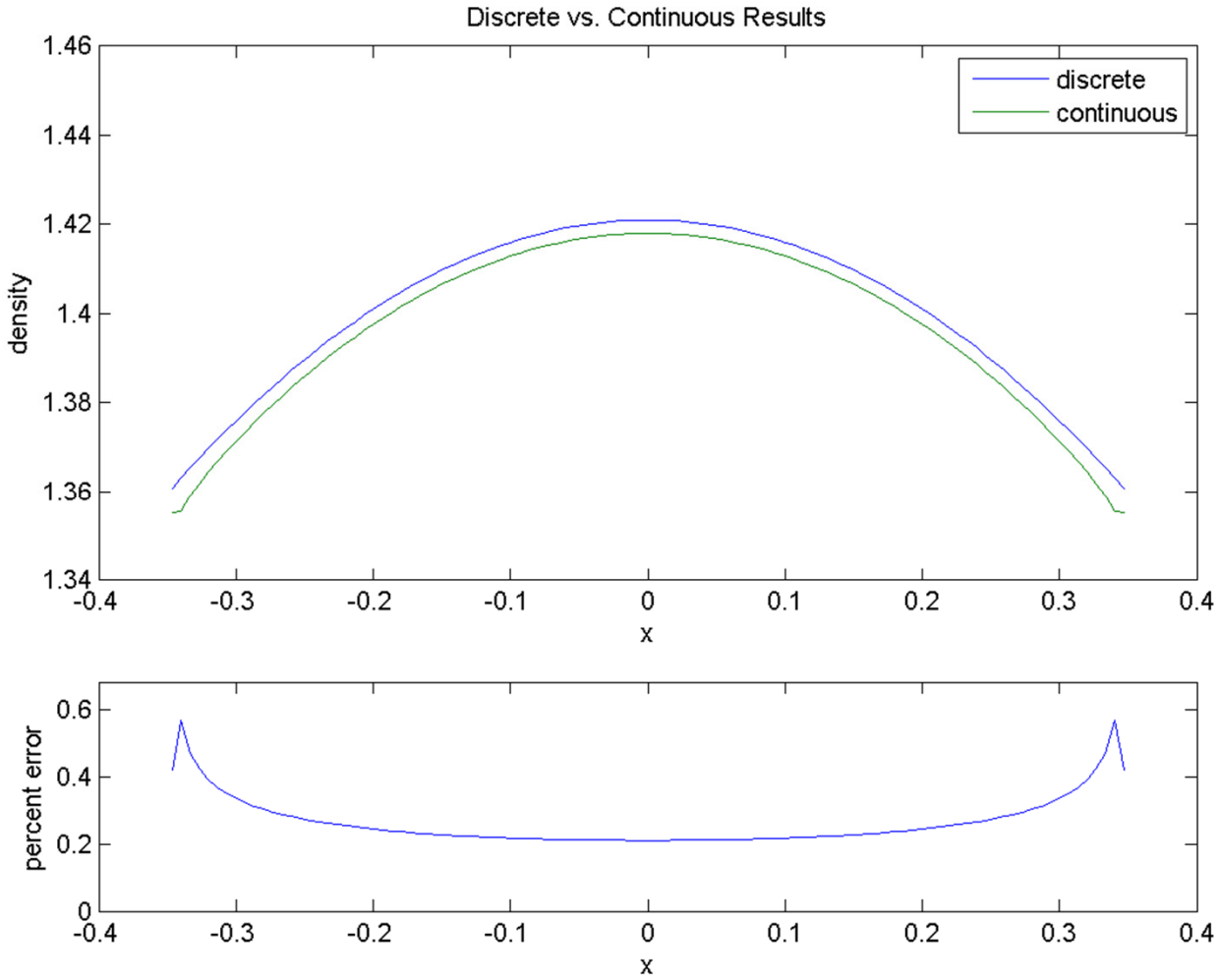
$$M(x) = \int_{-\infty}^{x} \rho(z)dz$$

- Sample the cumulative mass function,
  fit an interpolating polynomial,
  and differentiate to get discrete density

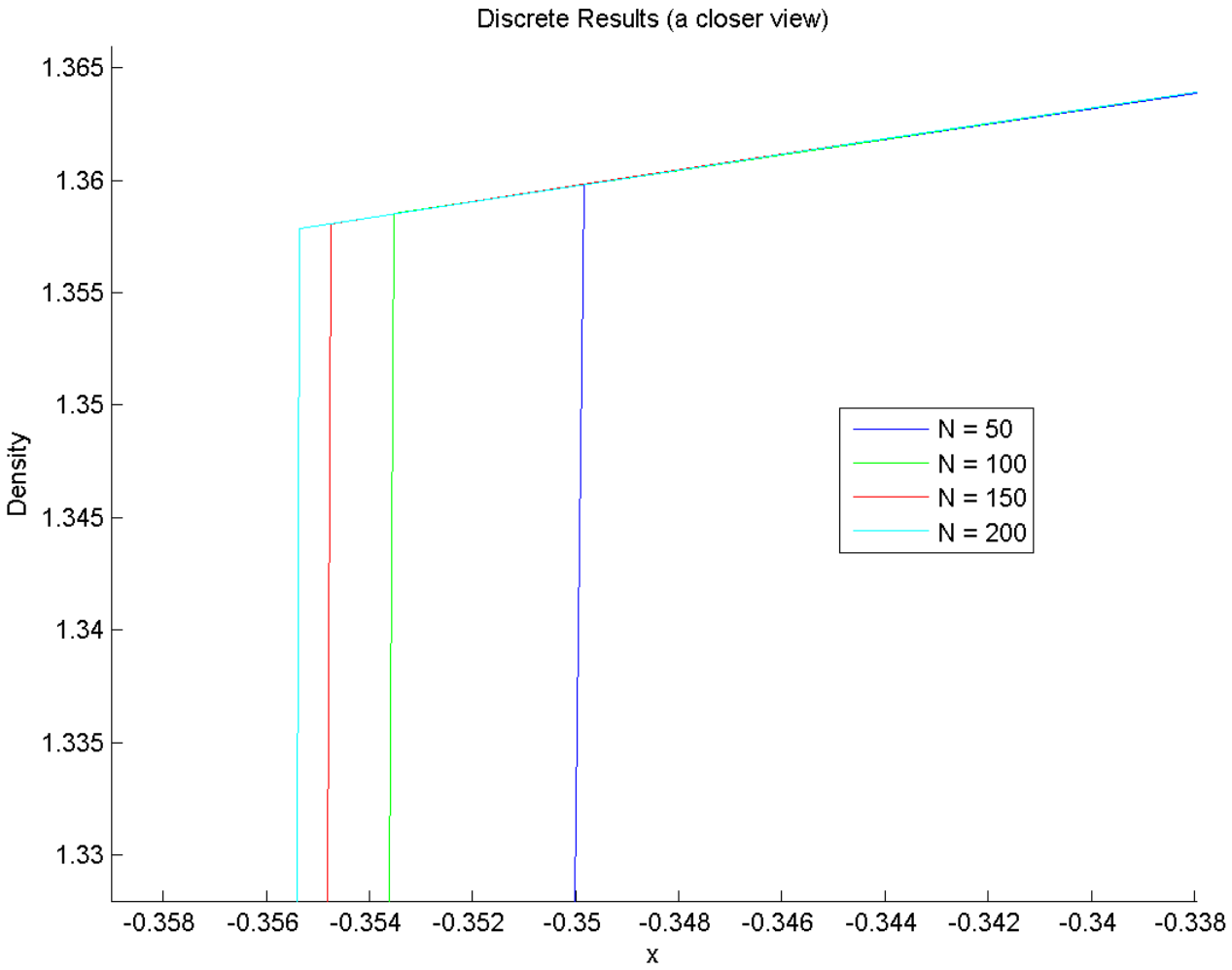
Results of Comparison



Results of Comparison



Refinement Testing



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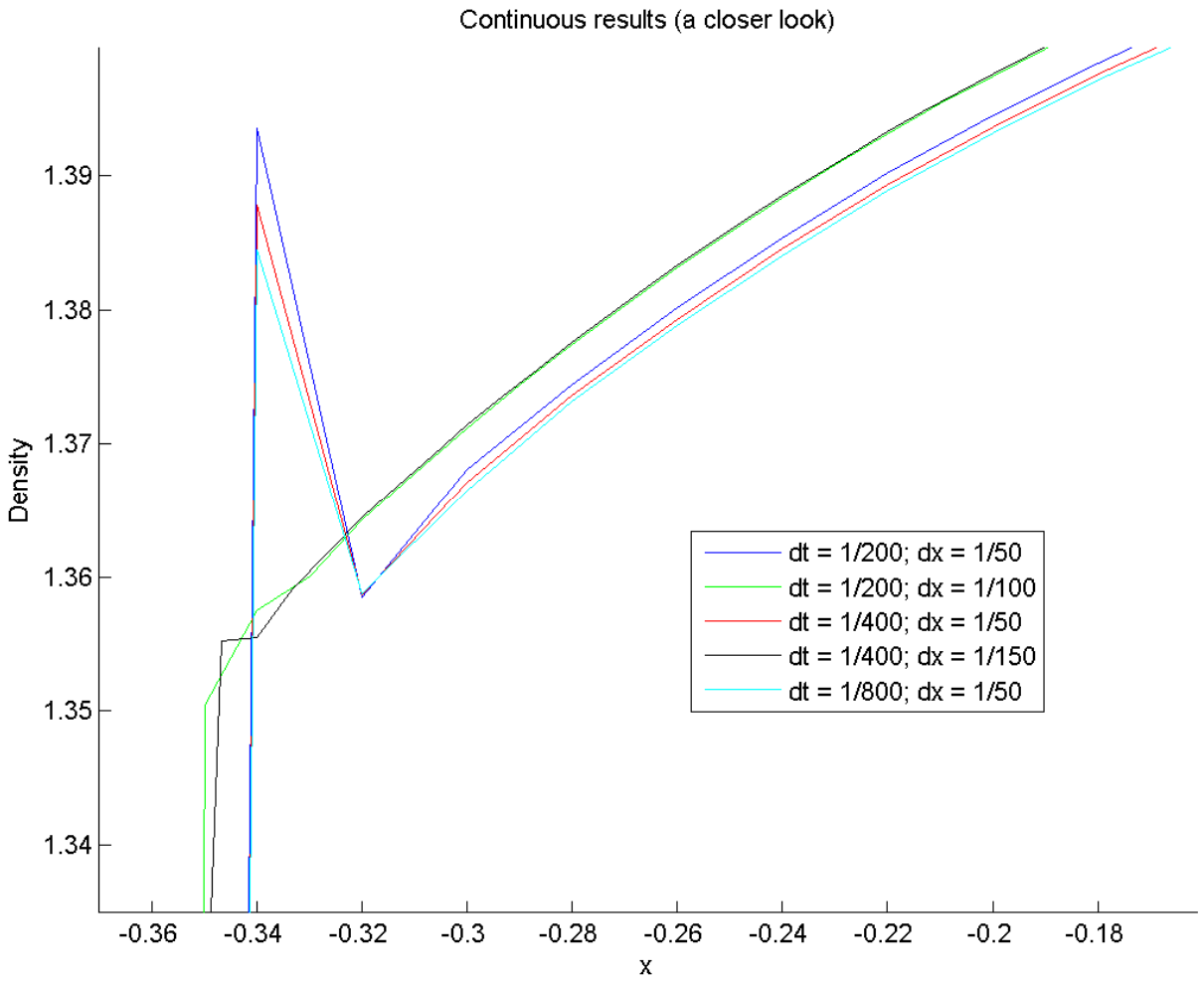

Future Work

- Improve numerical accuracy in continuous simulation
- Study behavior of model with different parameters
- Study continuous model analytically
  - Prove asymptotic behavior
  - Try to solve analytically (?)

Acknowledgment

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