Craig Weidert
Math 164 - Scientific Computing
Homework #6
Due May 4, 2007

Introduction

For this assignment, I chose to try my hand at simulating the paths of a bowling ball down a lane. I very much enjoy bowling so this seemed like a fine excuse to get credit for trying to improve my game. I have also been curious about exactly what does make a ball curve down a lane and how varying parameters of the ball, lane, etc can effect the paths of the ball and thus your score.

I used the excellent paper "What Makes Bowling Balls Hook" by Cliff Frohlich from the University of Texas, Austin as a guide on my quest. All of the equations in this paper except those pertaining to determining the moment of inertia are taken from this paper. To solve the equations to find the actual paths of the ball, I wrote a number of first-order differential equations and then solved them for the length of the lane using Matlab's ode45. In order to search over the initial parameters (direction, speed, rotation speed, position on the lane) to find those that would lead the pins to maximum destruction, I used Matlab's fminsearch function. I also used Matlab's fminsearch function in order to find parameters which corresponded to real-life bolwing balls.

Approach

A bowling lane is 42 inches wide and 60 feet from the foul line to the first pin. The coefficient of friction μ must never be more than .39. However, μ is generally as low as .04 for the first two thirds of the lane and .2 for the final, "un-oiled" third. At the end of the lanes are ten pins which are 15 inches tall, 4.7 inches wide, and 3.5 pounds. They are arranged in a triangle. The general idea behind bowling is to knock down as many pins as possible given two shots at them with the ball. Special bonuses are given if one clears all of the pins and gets a "strike" or "spare".

Bowling balls must weigh no more than 16 pounds and must have a radius between 8.5 and 8.595 inches. They don't have to be uniformly dense, but their radius of gyration must fall between 2.43 and 2.80 inches and its center of gravity generally must not be more than 1mm from the center of the ball. These non-uniformities are actually on purpose and give the ball a lot of character. For instance, simply by shifting the center of gravity of the ball, one can achieve a deflection of up to 10 cm.

It is easier to achieve higher scores if one gives the ball some spin and attacks the pins at an angle. This is because this method is less likely to leave splits, the most dreaded configuration of pins in bowling. It has been experimentally shown that the optimal angle of approach for the ball striking the pins is about six degrees and that the optimum place for them to strike the pins is a bit to the right of the of head pin (at least if you're right handed and hooking the ball to the left). So, for this project, I am going to determine initial parameters (such as initial linear velocity V_0, angluar velocity ω_0, direction of release, and point of release) so that the ball will collide with the pins in an optimal way. Default values for V_0 and ω_0 are 8.0 m/s and 30 rad/s.

Methods

I should like to simulate the path of the ball down the lane. I will use the equations taken from Frohlich's paper. We will work in the frame of the bowling lane where the positive x direction is down the lane, the positive y direction is left across the lane, and the positive z direction is up. We define the origin to be the point exactly the radius of the ball up from the right end of the foul line (the beginning of the lane). Let m be the mass and of the ball, R_{ball} be the radius of the ball, $\vec{r}$ be a vector giving the position of the center of mass of the ball, and $\vec{r}_\Delta$ be the vector from the center of mass of the ball to the center of the ball. Let $\vec{\alpha}$ and $\vec{\omega}$ be the rotational velocity and acceleration of the ball respecively.

The equations dictating the linear and rotational acceleration on the ball are given by the equations

$$m\vec{r}'' = \vec{F}_{\text{con}} + \vec{F}_{\text{g}} \tag{1}$$

$$\frac{d}{dt}(I\vec{\omega}) = (\vec{r}_\Delta + \vec{R}_{\text{con}}) \times \vec{F}_{\text{con}}, \tag{2}$$

where $\vec{R}_{\text{con}}$ is the vector from the center of the ball to the lane and $\vec{F}_{\text{con}}$ is the contact force on the ball due to friction, and I is the moment of inertia tensor. In the case that I is diagonal and $\vec{R}_\Delta$ is zero, Equation 2 simplifies to

$$I\vec{\alpha} = \vec{R}_{\text{con}} \times \vec{F}_{\text{con}} \tag{3}$$

but in general the situation is much more complicated. In the general case, we have

$$\frac{d}{dt}(I\vec{\omega}) = (I_0 + I_{\text{dev}})\vec{\alpha} + \vec{\omega} \times (I_{\text{dev}}\vec{\omega}), \tag{4}$$

where I_0 is a diagonal matrix with entries equal to the average of the diagonal elements of I and $I_{\text{dev}} = I - I_0$.

In Frohlich's paper, he derives the appropriate equations. We can get the slippage $\vec{s}$ of the ball with the equation

$$\vec{s} = \vec{R}_{\text{con}} \times \vec{\omega} - \vec{r}' \tag{5}$$

and depending on whether the ball is slipping or not, use the appropriate equations.

If the ball is sliding on the lane, then we have

$$(I_0 + I_{\text{dev}} + I^s_\Delta + I^s_{\Delta\Delta})\vec{\alpha} = \vec{\tau}_{\text{fric}} + \vec{\tau}_{\text{dev}} + \vec{\tau}^s_\Delta + \vec{\tau}^s_{\Delta\Delta} \tag{6}$$

where

$$I^s_\Delta = (R_{\text{ball}}) \begin{bmatrix} r_{\Delta y}\mu_y & -r_{\Delta x}\mu_y & 0 \\ r_{\Delta y}\mu_x & r_{\Delta x}\mu_x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{7}$$

$$I^s_{\Delta\Delta} = \begin{bmatrix} r_{\Delta y}(r_{\Delta y} - r_{\Delta z}\mu_y) & -r_{\Delta x}(r_{\Delta y} - r_{\Delta z}\mu_y) & 0 \\ r_{\Delta y}(r_{\Delta z}\mu_x - r_{\Delta x}) & -r_{\Delta x}(r_{\Delta z}\mu_x - r_{\Delta x}) & 0 \\ r_{\Delta y}(r_{\Delta x}\mu_y - r_{\Delta y}\mu_x) & -r_{\Delta x}(r_{\Delta z}\mu_y - r_{\Delta y}\mu_x) & 0 \end{bmatrix}, \tag{8}$$

$$\vec{\tau}_{\text{fric}} = g\vec{R}_{\text{con}} \times \vec{\mu} \tag{9}$$

$$\vec{\tau}_{\text{dev}} = (I_{\text{dev}}\vec{\omega}) \times \vec{\omega} \tag{10}$$

$$\vec{\tau}^s_{\Delta} = g\vec{r}_{\Delta} \times \vec{\mu} + a_\omega \vec{R}_{\text{con}} \times \vec{\mu}, \tag{11}$$

$$\vec{\tau}^s_{\Delta\Delta} = a_\omega \vec{r}_{\Delta} \times \vec{\mu}, \tag{12}$$

$\vec{F}_{\text{con}} = F_n\vec{\mu}$, and a_ω is the acceleration in the vertical direction due to the ball spinning around. If the ball is rolling we have

$$(I_0 + I_{\text{dev}} + I_{\text{roll}} + I^r_{\Delta})\vec{\alpha} = \vec{\tau}_{\text{dev}} + \vec{\tau}^r_{\Delta} + \vec{\tau}^r_{\Delta\Delta} \tag{13}$$

where

$$I_{\text{roll}} = \begin{bmatrix} R^2_{\text{ball}} & 0 & 0 \\ 0 & R^2_{\text{ball}} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{14}$$

$$I^r_{\Delta} = \begin{bmatrix} r_{\Delta y}^2 + r_{\Delta z}^2 - 2r_{\Delta z}R_{\text{ball}} & -r_{\Delta x}r_{\Delta y} & -r_{\Delta x}r_{\Delta z} + r_{\Delta x}R_{\text{ball}} \\ -r_{\Delta x}r_{\Delta y} & r_{\Delta x}^2 + r_{\Delta z}^2 - 2r_{\Delta z}R_{\text{ball}} & -r_{\Delta y}r_{\Delta z} + r_{\Delta y}R_{\text{ball}} \\ -r_{\Delta x}r_{\Delta z} + r_{\Delta x}R_{\text{ball}} & -r_{\Delta y}r_{\Delta z} + r_{\Delta y}R_{\text{ball}} & r_{\Delta x}^2 + r_{\Delta y}^2 \end{bmatrix}, \tag{15}$$

$$\vec{\tau}^r_{\Delta} = g\vec{r}_{\Delta} \times \vec{n} + R_{\text{ball}}[\omega^2 r_{\Delta} \times \vec{n} + (\vec{\omega}\vec{r}_{\Delta})\vec{n} \times \omega], \tag{16}$$

and

$$\vec{\tau}^r_{\Delta\Delta} = (\vec{\omega}\vec{r}_{\Delta})\omega \times \vec{r}_{\Delta}. \tag{17}$$

Say we have N particles where the mass of the i^{th} particle is m_i and the positon is

(x_i, y_i, z_i). The center of mass is then

$$(\sum_{i=1}^{N} m_i x_i, \sum_{i=1}^{N} m_i y_i, \sum_{i=1}^{N} m_i z_i) / \sum_{i=1}^{N} m_i \tag{18}$$

and the radius of gyration is given by

$$\sqrt{\frac{1}{N}\sum_{i=1}^{N}(r_i - r_{\text{avg}})^2}. \tag{19}$$

The moment of inertia tensor is given by

$$\begin{bmatrix} \sum_{i=1}^{N} m_i(y_i^2 + z_i^2) & -\sum_{i=1}^{N} m_i x_i y_i & -\sum_{i=1}^{N} m_i x_i z_i \\ \sum_{i=1}^{N} m_i x_i y_i & \sum_{i=1}^{N} m_i(x_i^2 + z_i^2) & \sum_{i=1}^{N} m_i y_i z_i \\ \sum_{i=1}^{N} m_i x_i z_i & \sum_{i=1}^{N} m_i y_i z_i & \sum_{i=1}^{N} m_i(x_i^2 + y_i^2) \end{bmatrix}. \tag{20}$$

Since bowling balls have a heavier, offset core, I modeled the bowling ball as an offset, denser sphere within another sphere. I varied the parameters for the radius of the inner sphere, the offset of the inner sphere, and the ratios of the densities of the inner and outer cores. I did so until I found that the center of mass offset and radius of gyration were exactly the that I wanted (1mm and 7.11cm). I used fminsearch to find these parameters and translated this into an appropriate moment of inertia tensor for the rest of the project.

What I essentially did was write everything in terms of first-orde differential equations, code up all of the equations in Matlab, and use ODE 45 to solve.

Results

My first result (conceptually, not in terms of my actual timeline) was finding parameters for the bolwing ball itself so I could get an appropriate moment of inertia tensor. I found that the core radius should be 4.12cm, the ratio of the densities should be 9.56, and the offset of the inner sphere should be 0.31cm. Using these values, I could calculate the moment of inertia tensor for use in the differential equations. I found that the moment of inertia tensor turned out to be a diagonal matrix that was roughly 0.0178 times the identiy matrix with the first entry being slightly less than the other two.

I found that no matter where on the lane I started the ball, I could come up with initial an initial direction and angular and linear velocities to lead the ball into the pins at the correct angle and position. I generally started fminsearch with initial linear and angular velocities around 8.0 m/s and 30 rad/s which are generally accepted values for bowlers who bowl around 200. I found that I could also drop the angular velocities to around 15 rad/s and still get fine results. Some example paths are shown in Table and Figure 1.

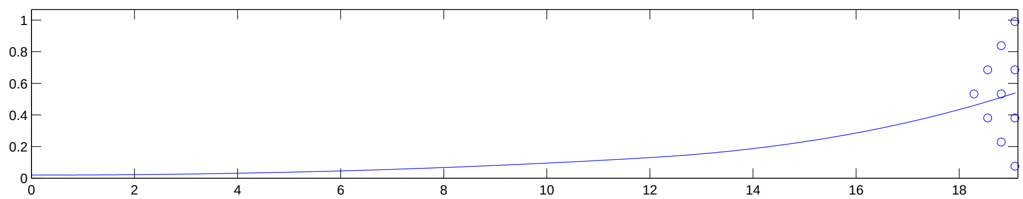
I also ran a number of tests with the edges of the lanes sharing the same high coefficient of friction as the end of the lane. During these tests, the ball deflected much more, which makes perfect sense.

Conclusions and Future Work

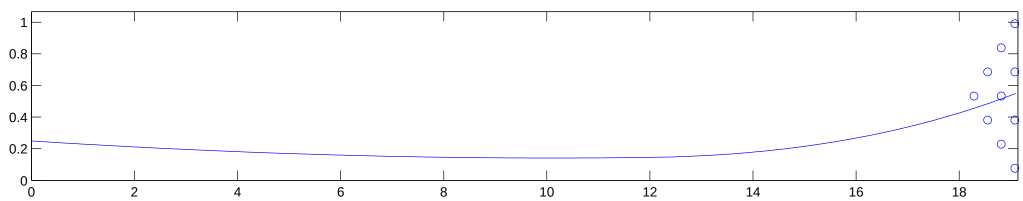
The results I obtained are very reasonable and seem to very closely mimic those which I see during actual bowling. I have often varied all four of the initial parameters from this study and have gotten strikes from many different angles.

I believe that my moment of inertia tensor is still a little off and could use a little more work. The reason I think this is because if I reduce the offset of the center of mass, it has less effect than was cited in Frohlich's paper. I think that this may be partially due to my sphere within a sphere model for bowling balls, which may be overly simplistic. In actual bowling balls, the core of a bowling ball is not necessarily a sphere and could any number of shapes. This would result in a moment of inertia tensor that was not necessarily diagonal.

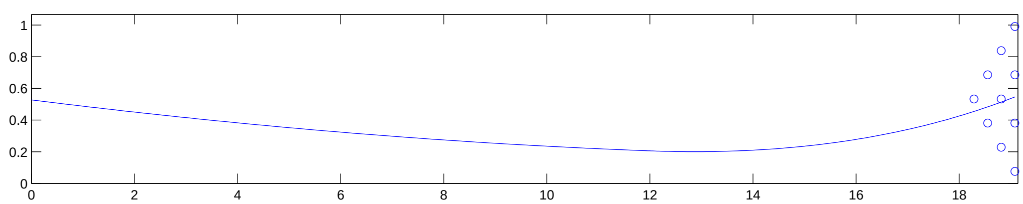
Some other useful future results could be obtained by varying the parameters of the ball and the lanes to see what interesting effects they have on the ball's path. Also interesting would have been to add some sort of error per time step in the ball or in the



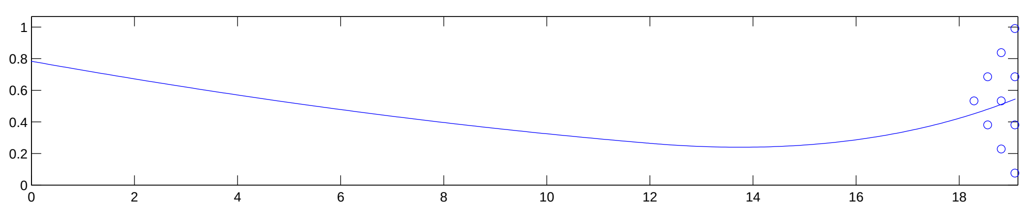
(a) Path 1: The ball on a path of destruction from the extreme right of the lane ($v_0 = 9.12$ m/s, $\omega_0 = 27.98$ rad/s).



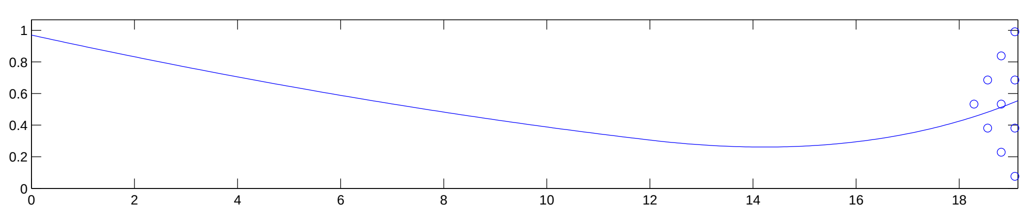
(b) Path 2: The ball heading from the mid-right of the lane ($v_0 = 8.46$ m/s, $\omega_0 = 28.86$ rad/s).



(c) Path 3: The ball bringing the heat form the center of the lane ($v_0 = 8.32$ m/s, $\omega_0 = 32.01$ rad/s). This is the approach I generally use when bowling.



(d) Path 4: The ball starting to the left of the lane ($v_0 = 8.03$ m/s,$\omega_0 = 30.83$ rad/s).



(e) Path 5: The ball from an initial position at the left of the lane ($v_0 = 7.91$ m/s, $\omega_0 = 31.78$ rad/s).

Figure 1: Paths of the ball from various points on the lane.

Path	θ_0	y_0	v_0	ω_0
1	-0.0002	0.0200	9.1256	27.9764
2	-0.0207	0.2488	8.4558	28.8638
3	-0.0405	0.5272	8.3274	32.0118
4	-0.0580	0.7836	8.0316	30.8325
5	-0.0710	0.9696	7.9076	31.7811

Figure 2: Some example results for initial parameters which led to optimal pin striking. I should note that y_0, the initial ball position is out of 1.0668.

parameters themselves to see what sorts of parameters yielded the most stable paths to the optimal striking position. This work would have been the most useful.