Modeling Electrons in an Electrostatic Analyzer

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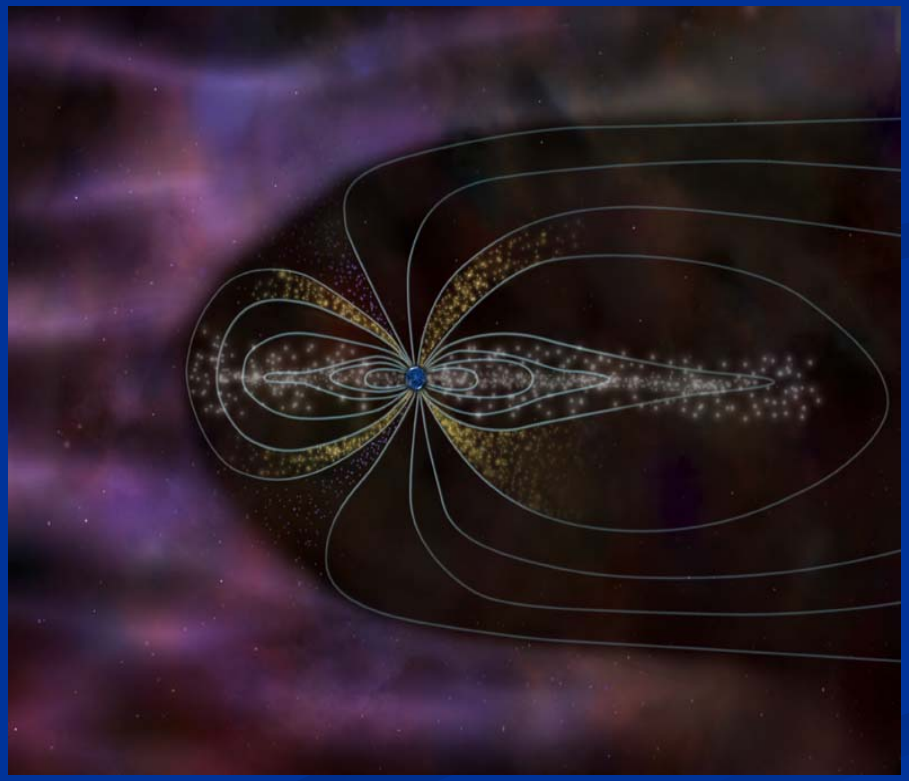
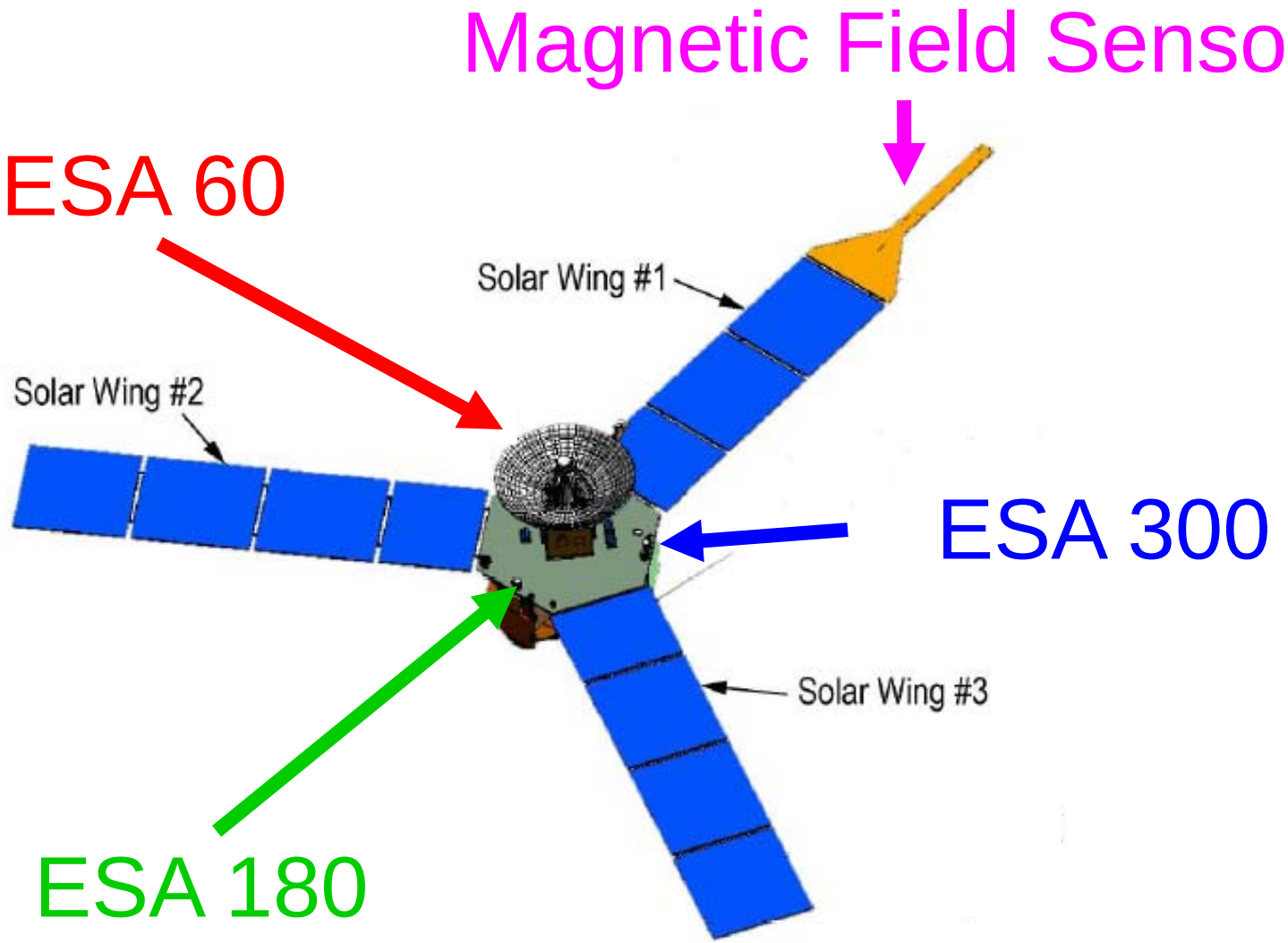


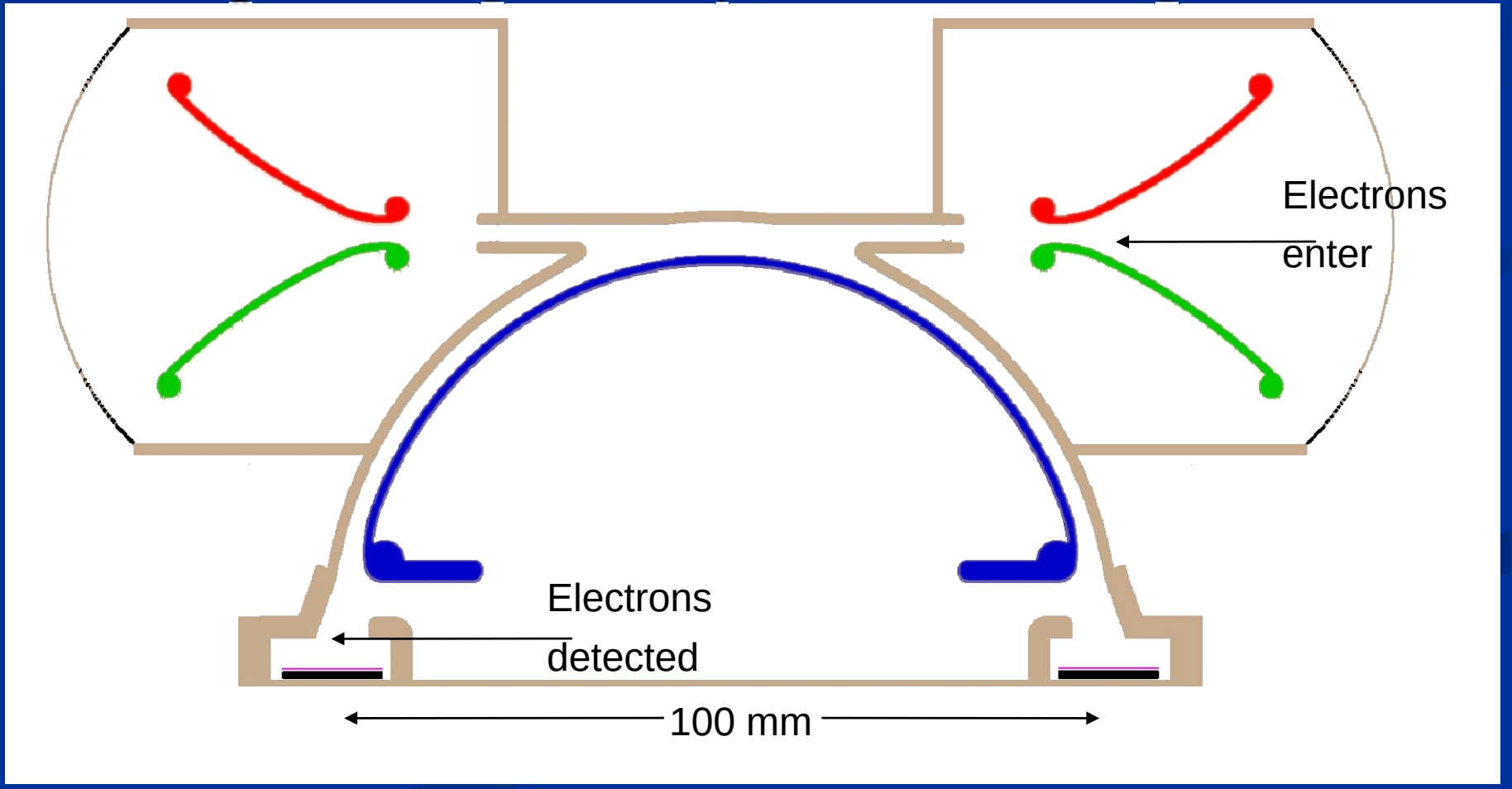
Based on a clinic project sponsored by SwRI.

JUNO Satellite



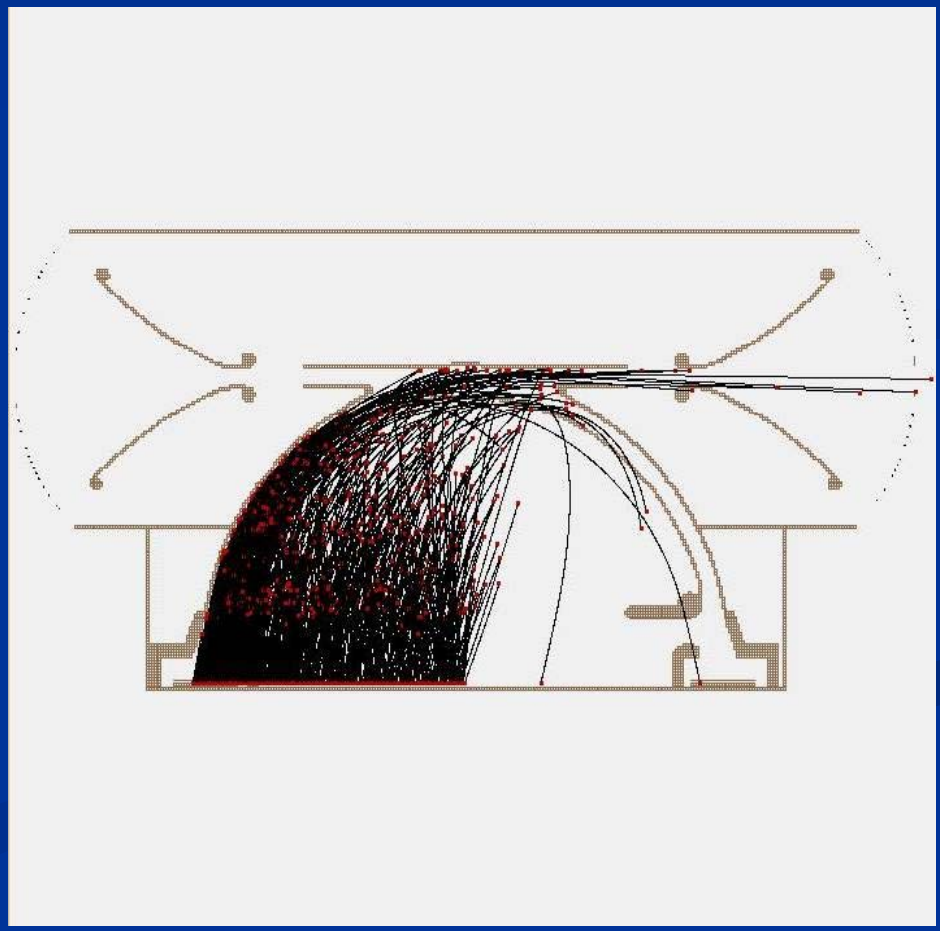
Electrostatic Analyzer

- Measures velocity distribution of electrons
- Complicated geometry, several voltages



Numerical Simulations

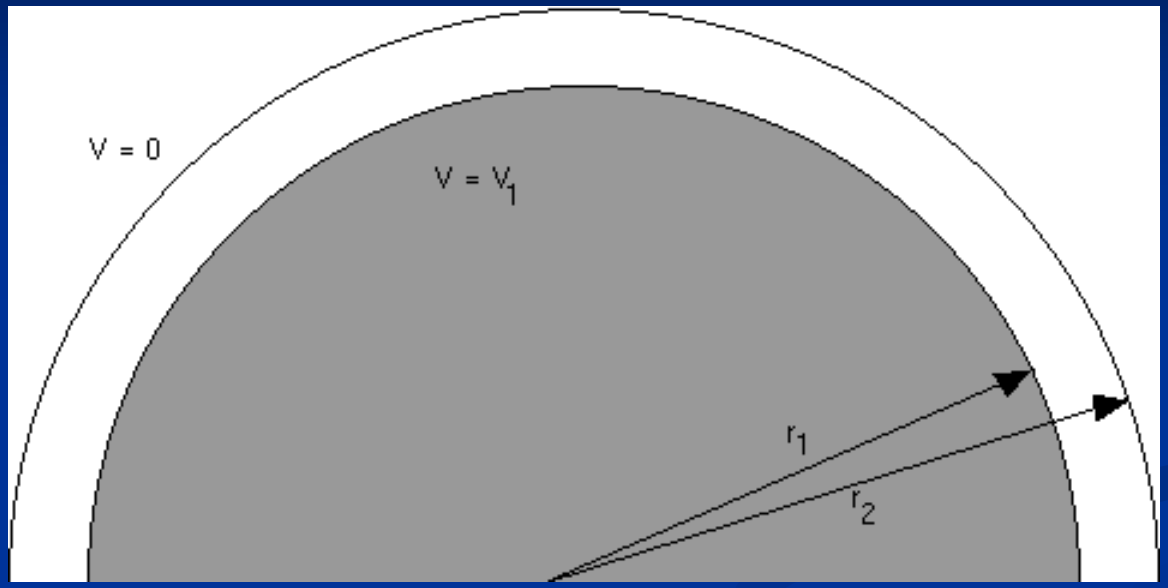
- Starting point for analysis
- SIMION software
  - Solves for E-fields
  - Launches particles



Scientific Computing Project

- Model a simplified ESA numerically
- Launch particles through the ESA
- Compare energies of electrons detected to those from theory or from SIMION

Simplest ESA



- 2D problem
- Inner hemispherical plate
- Outer hemispherical plate grounded
- Electrons ‘detected’ if they can go around the circle

Analytical results

- Assuming the gap is small, the field is constant

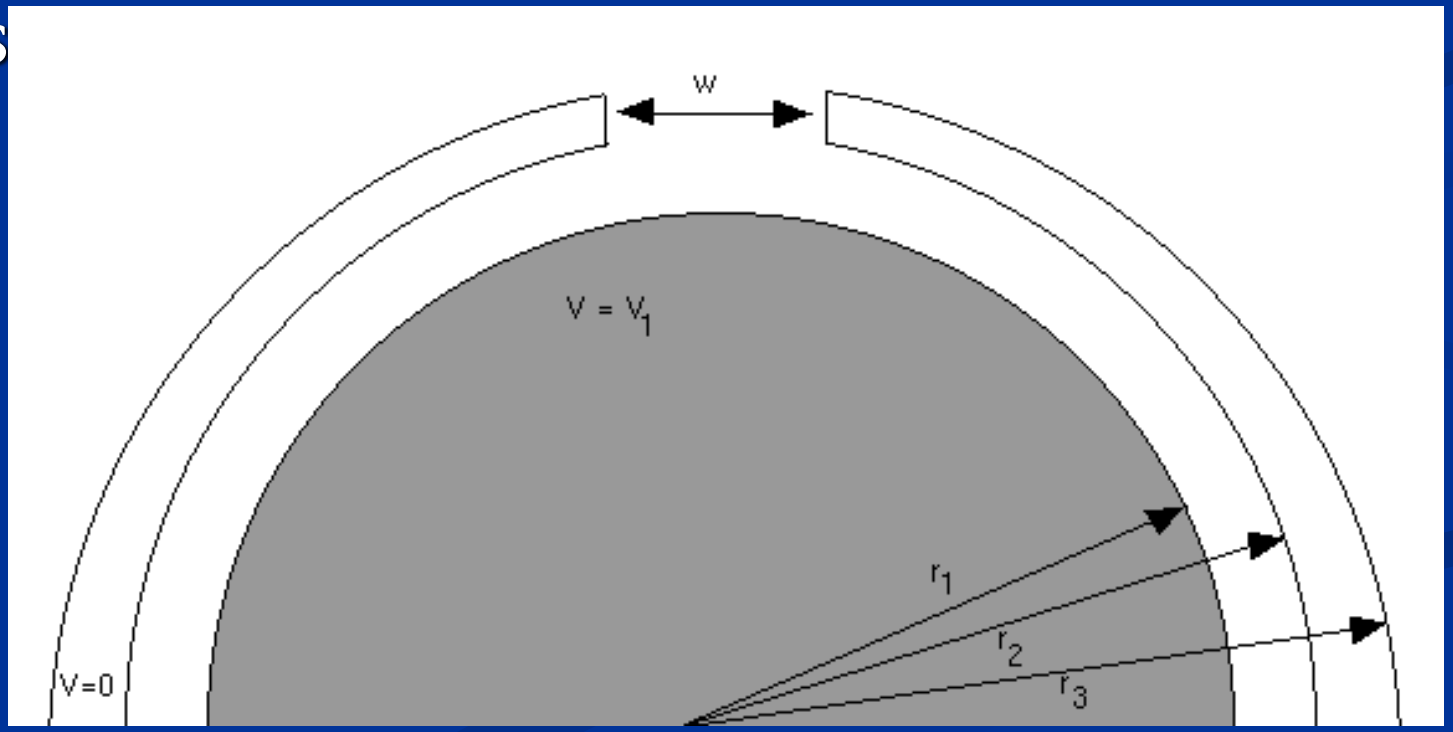
$$\vec{E} = \frac{V}{r_2 - r_1}\hat{r}$$

- Balance centripetal force and force on electron
- To be detected, electron must have kinetic e

$$T = \frac{\bar{r}qV}{2(r_1 - r_2)}$$

Non-ideal ESA

- A gap makes the problem more interesting!
- Outer plate has a thickness and a gap
- Electrons have to get through the gap to be s



Algorithms

- Method of relaxation to solve Laplace's equation and calculate V
  - Discretize space – currently on a cartesian grid
  - Set boundary condition voltages
  - Set the potential at each point to be the average of the points next to it, repeat until changes are small.

Algorithms

- Calculating E-field by taking the gradient of V
  - Second-order accurate centered difference formulas used in interior
  - First-order formulas would be used on edges
    - Doesn't matter – V is fixed to 0 on outside edges
  - E-field interpolated linearly between gridpoints

Algorithms

- Trajectory calculation
  - Leapfrog method (second-order accurate)

$$x_{n+1} = x_n + v_n dt + \frac{1}{2} a_n dt^2$$

$$v_{n+1} = v_n + \frac{a_n + a_{n+1}}{2} dt$$

Current status

- Algorithms implemented and tested separately
  - Given exact field, error in trajectories is small
  - Given a grid and voltages, I can see fringe fields
- Future work
  - Put it all together
  - Improve accuracy and efficiency - polar coordinates?
  - Make pretty pictures