Density and Elasticity

Corey Hebert

Math 164 – Scientific Computing

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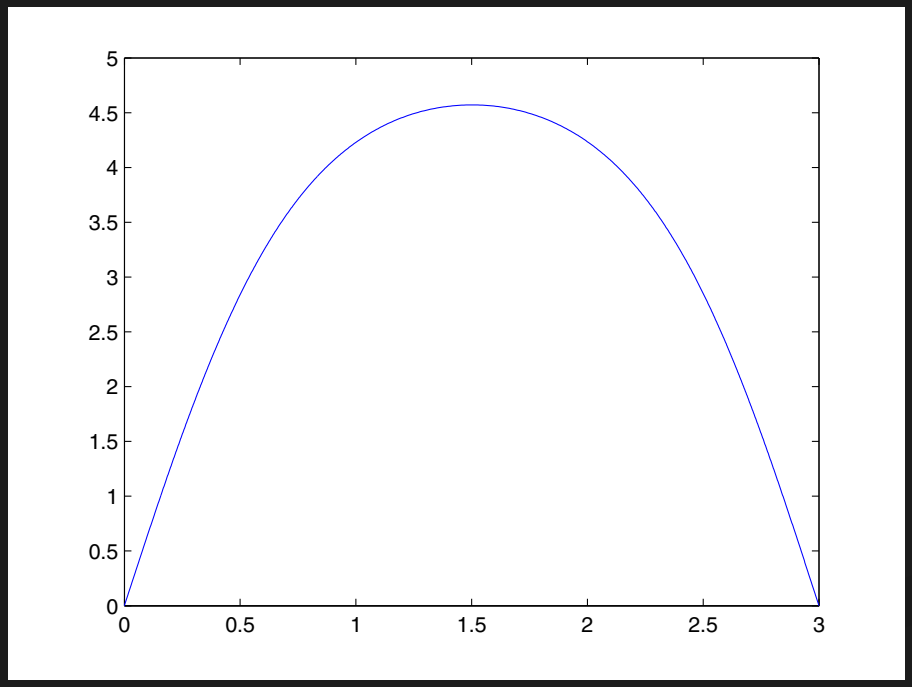
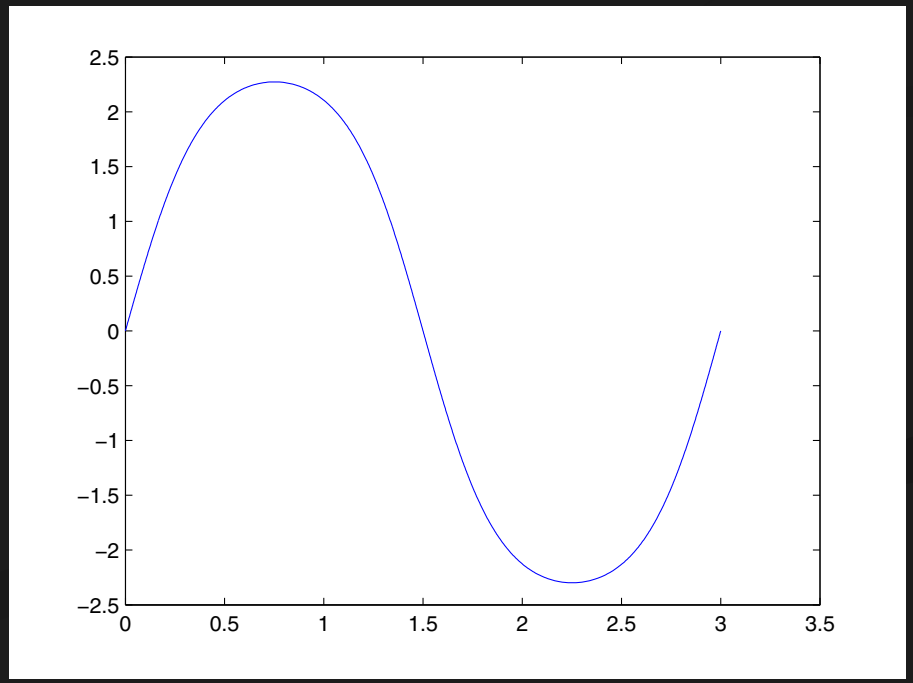
Homework 4

$$x' = \cos(\theta)$$

$$y' = \sin(\theta)$$

$$\theta'' = -2(\theta')^2 \tan(\theta) + \frac{\rho'(s)}{\rho(s)}\theta' - \frac{\omega^2 \rho(s)}{C}\sin(\theta)\cos^2(\theta)$$





Physical Explanation

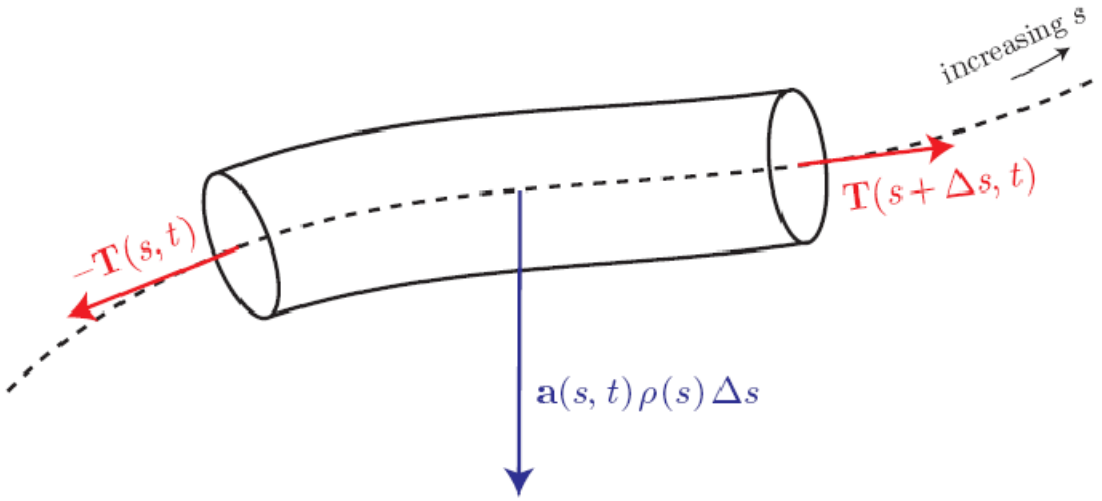


Fig. 1 *Free-body diagram of a small segment of the curve. The dashed curve is* $\mathbf{x}(s,t)$*, the centerline of the string, rope, or chain.*

$$\mathbf{x}_{tt}(s,t)\rho(s)\Delta s = -\mathbf{T}(s,t) + \mathbf{T}(s-\Delta s,t) - \mathbf{a}(s,t)\rho(s)\Delta s.$$

$$\rho(s)\mathbf{x}_{tt}(s,t) = \mathbf{T}_s(s,t) + \mathbf{a}(s,t)\rho(s).$$

General Equations

Inextensible :

$$\mathbf{0} = \frac{\partial}{\partial s}\left(T(s)\mathbf{x}_s\right) + \omega^2 y(s)\rho(s)\mathbf{j}$$

Extensible :

$$\mathbf{0} = \frac{\partial}{\partial s}\left(k\left(\|\mathbf{x}_s\| - 1\right)\frac{\mathbf{x}_s}{\|\mathbf{x}_s\|}\right) + \omega^2 y(s)\rho(s)\mathbf{j}$$

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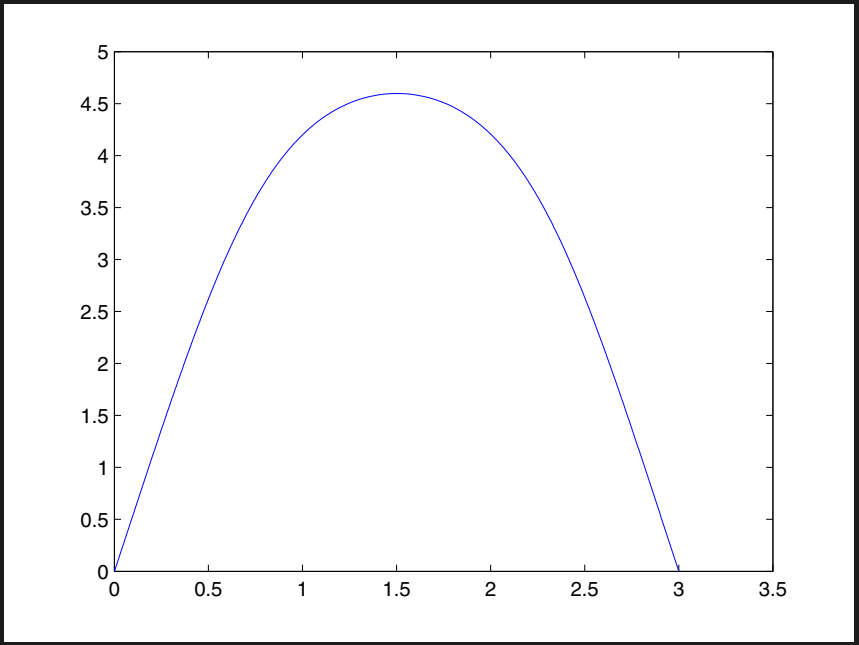
Linearly Elastic

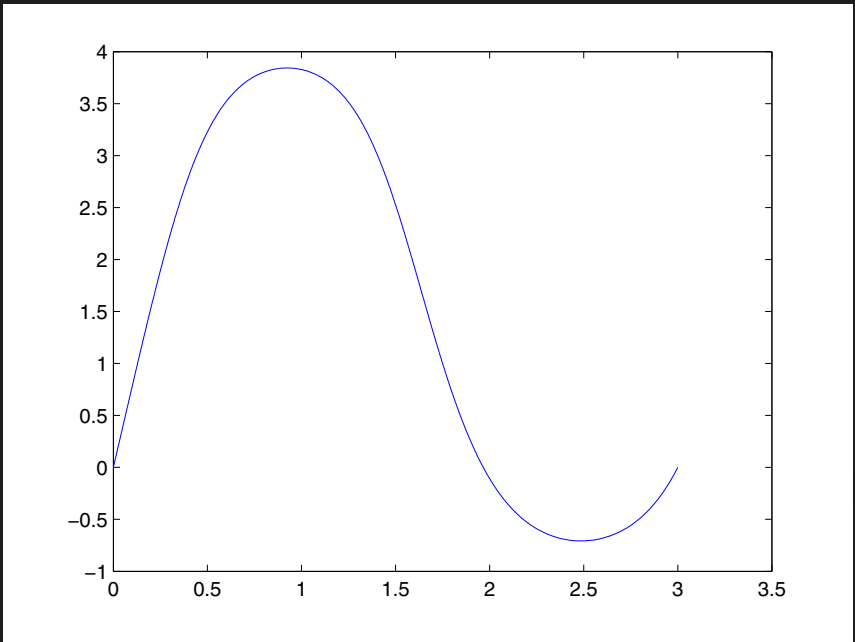
Mathematica

$$\left\{\left\{x''[s] \to \frac{-\text{omega}^2\ \text{rho}\ y[s]\ x'[s]^2 + k'[s]\ \left(y'[s]^3 - x'[s]\ y'[s]^2\ \sqrt{x'[s]^2 + y'[s]^2} - x'[s]^3\ \left(-1 + \sqrt{x'[s]^2 + y'[s]^2}\right)\right)}{k[s]\ \left(x'[s]\ y'[s] + y'[s]^2\ \sqrt{x'[s]^2 + y'[s]^2} + x'[s]^2\ \left(-1 + \sqrt{x'[s]^2 + y'[s]^2}\right)\right)},\right.\right.$$

$$\left.\left.y''[s] \to -\frac{\text{omega}^2\ \text{rho}\ y[s]\ \left(x'[s]\ y'[s] + x'[s]^2\ \sqrt{x'[s]^2 + y'[s]^2} + y'[s]^2\ \sqrt{x'[s]^2 + y'[s]^2}\right) + k'[s]\ y'[s]\ \left(x'[s]\ y'[s] + x'[s]^2\ \left(-2 + \sqrt{x'[s]^2 + y'[s]^2}\right) + y'[s]^2\ \left(-1 + \sqrt{x'[s]^2 + y'[s]^2}\right)\right)}{k[s]\ \left(x'[s]\ y'[s] + y'[s]^2\ \sqrt{x'[s]^2 + y'[s]^2} + x'[s]^2\ \left(-1 + \sqrt{x'[s]^2 + y'[s]^2}\right)\right)}\right\}\right\}$$

Variable Density Results?






```
>> script(2, @(s) sin(s*pi/10), 3, 10, [1.2; 6])

x =
   1.3906
   4.0576
```

```
>> script(2, @(s) sin(s*pi/10), 3, 10, [1.2; 2])

x =
   1.4434
   1.6210
```

Pending

- Fix Variable Density Case
- Experiment with Elasticity