Modeling an Elastic Jump Rope

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Math 164

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Questions about an Elastic Jump Rope

- Are there any unexpected differences from the inextrisible case?
- Where along the rope does the most of the stretching occur?
- How does a variable mass density affect the shape and stretching?

Mathematical Model

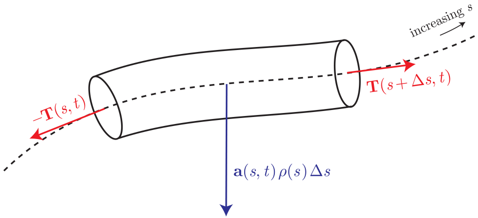


Fig. 1 *Free-body diagram of a small segment of the curve. The dashed curve is* $\mathbf{x}(s,t)$*, the centerline of the string, rope, or chain.*

General Wave Equation

This equation describes the motion of a one-dimensional object.

$$\rho(s)\mathbf{x}_{tt}(s,t) = \mathbf{T}_s(s,t) + \mathbf{a}(s,t)\rho(s)$$

Mathematical Model

For our elastic jumprope:

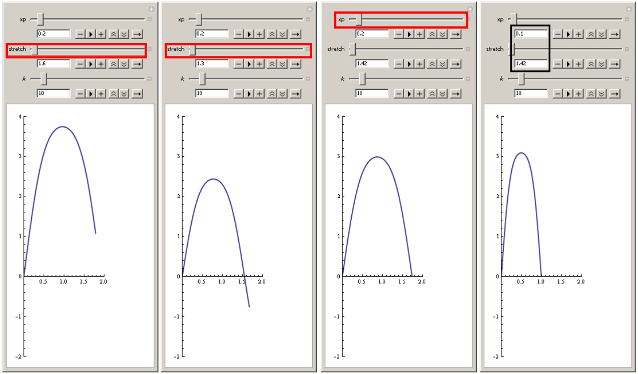
- The shape is static in the rotating plane
- The material is linearly elastic
- The centrifugal force dominates

Equations of Motion for the Jumprope

From the general wave equation we get the BVP

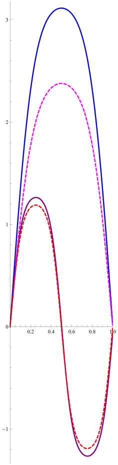
$$0 = \frac{\partial}{\partial s}\left(\frac{k\mathbf{x}_s(||\mathbf{x}_s|| - 1)}{||\mathbf{x}_s||}\right) - \omega^2\rho(s)y(s)\hat{\mathbf{j}}$$

$$x(0) = y(0) = 0, x(L) = H, y(L) = 0.$$



- Approximate the solution graphically by varying initial conditions
- Use these approximate initial conditions in a shooting method

Basic Results

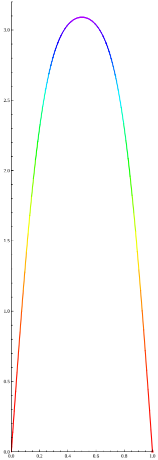


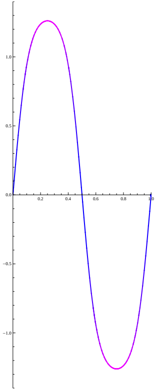
Comparison to Inextensible Jumprope

Additional Observations

- Increasing angular velocity increases the stretching of the jumprope
- Jumpropes with higher mass density stretch more
- Increasing the "spring constant" makes the jumprope behave more like the inextensible case

The Stretching of the Jumprope

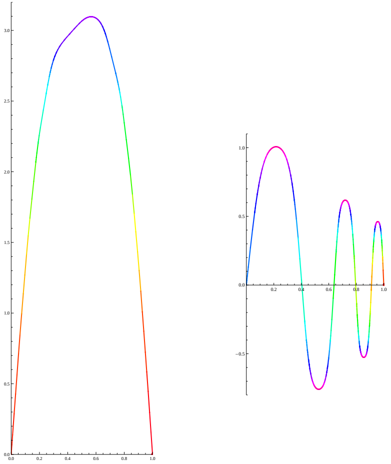




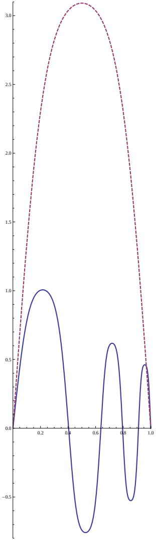
Constant ρ

The Stretching of the Jumprope

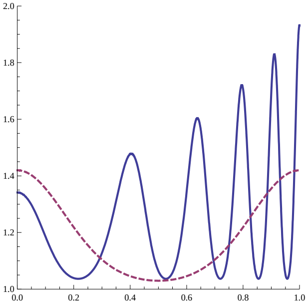
Constant ρ



Non-constant ρ



Jump Rope Shapes



Stretch Plot