WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION

GOOD WRITING

Problem A1

A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?

- Consider a <u>cross-section</u> of this cone & cube along the diagonal of the base of the cube. It looks like:

$s\sqrt{2}$ is the length of diagonal of top of cube

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S is the length of the side of cube

- We wish to <u>find</u> s.

- Since the triangle ABC and the shaded $\triangle$ are <u>similar</u>, we have:

$$\frac{1}{3} = \frac{1 - \frac{s\sqrt{2}}{2}}{s}$$

Thus $\quad s = 3 - \frac{3\sqrt{2}}{2} s$

Hence $\quad s\left(1 + \frac{3\sqrt{2}}{2}\right) = 3$

which, when solved for s, yields

$$S = \frac{9\sqrt{2} - 6}{7}.$$

the really trivial calculations are omitted, but just enough detail is included, so reader can follow argument.

If you graded this, it would be easy to follow and check to see the main ideas are correct!
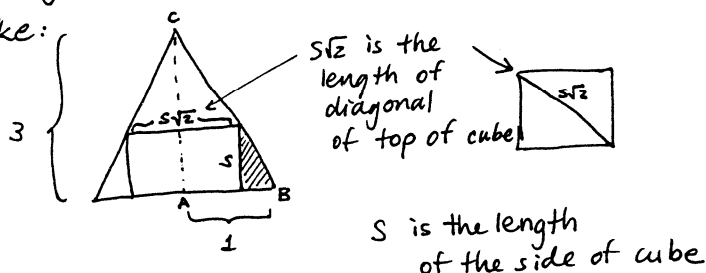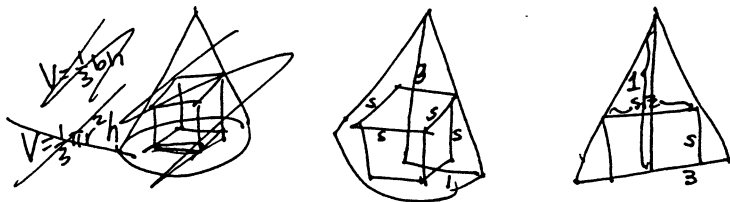
(compare with other side)
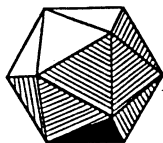
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<u>Problem A1</u>

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$V = \frac{1}{3}6h$

$V = \frac{1}{3}\pi r^2 h$

The cube is in the cone, and we can see that the bottom of the cone is the bottom of the cube so the top of the cone ~~is the~~ touches at two points. So the two triangles are similar meaning $\frac{1}{3} = \frac{1 - \frac{s\sqrt{2}}{2}}{s}$ because $s\sqrt{2}$ is the diagonal of the cube. Writing in one big long paragraph is very hard for the grader to read, so I know the grader will love me. So then the side-length of the cube ~~s~~ can be found.

$$S = 3 - \frac{3\sqrt{2}}{2}S \qquad S + \frac{3\sqrt{2}}{2}S = 3$$

$$s\left(1 + \frac{3\sqrt{2}}{2}\right) = 3$$

$$S = \frac{3}{1 + \frac{3\sqrt{2}}{2}}$$

$$\frac{3}{1 + \frac{3\sqrt{2}}{2}}$$

$$\frac{s\sqrt{2}}{3 + \sqrt{2}} \times \frac{3 - \sqrt{2}}{3 - \sqrt{2}}$$

$$(3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 2 = 7$$

$$(3\sqrt{2})(3 - \sqrt{2}) = 9\sqrt{2} - 6$$

$$\boxed{\dfrac{9\sqrt{2} - 6}{7}}$$