Additional Problems for HW #1

For following the two curvilinear coordinate systems,

I. $x = \frac{1}{2}(u^2 - v^2)$; $y = uv$ and
II. $x = v - u^3$; $y = u + v$,

(a) sketch some isolines of u and v (curves corresponding to constant u or constant v) in the x-y plane and show the region of the x-y plane that corresponds to the region $0 < u < 1$, $0 < v < 1$,

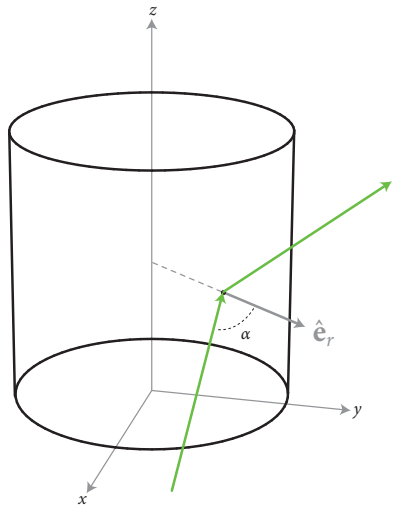
(b) figure out if the the curvilinear coordinate system is orthogonal,

(c) calculate the Jacobian matrix,

(d) find the locations in the u-v plane at which the determinant of the Jacobian is zero (these are locations at which the coordinate system is not locally one-to-one),

(e) calculate the area of the region from part (a).

III. A laser beam shines on the unit cylinder parallel to the z-axis at the point $r = 1$, $\theta = \pi/3$, $z = 1$ (in polar coordinates). The laser beam has incident direction $\mathbf{d} = -\mathbf{i} + \mathbf{k}$. What is the angle of incidence, α, as shown on the diagram below? (Extra credit: What is the direction of the beam after being reflected?)



IV. Describe, using words and pictures, what the vector field

$$\mathbf{F}(x, y, z) = -\frac{x}{(x^2 + y^2 + z^2)^{3/2}}\mathbf{i} - \frac{y}{(x^2 + y^2 + z^2)^{3/2}}\mathbf{j} - \frac{z}{(x^2 + y^2 + z^2)^{3/2}}\mathbf{k}$$

looks like. Write this vector field more succinctly using a different coordinate system.