HW #5: Surface Integral Problems

1. Use a surface integral to compute the lateral surface area (i.e. not including the top and bottom discs) of a cylinder with radius a and height h. Use geometric reasoning to determine the area of a small surface element.

2. Compute the surface integral of the vector field $\mathbf{F} = x\mathbf{i}$ over these surfaces:

   (a) a sphere of radius a centered at the origin,

   (b) a cylinder of radius a, height h, centered at the origin (including the top and bottom discs), parallel to the z-axis, and

   (c) a cube with side lengths $2L$, centered at the origin.

   Hint: To save work, first think about which sides of the cube and cylinder will actually contribute to the surface integral.)

3. Compute the flux of the vector field $\mathbf{F} = x^2\mathbf{i} + z\mathbf{k}$ through the portion of the sphere $x^2 + y^2 + z^2 = 1$ that is above $z = 1/2$.

4. Let X be the surface of an infinitely long cylinder of radius a, parallel to the y-axis. Think of at least three different (nonzero) vector fields for which the flux through X is zero. Explain why the flux is zero in each of your examples.

5. Consider a charge q placed at the origin. The electric field due to this charge is

$$\mathbf{E} = q\frac{\hat{\rho}}{\rho^2} = q\frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{3/2}}.$$

   (The ρ above is the radius variable in spherical coordinates.)

   (a) Compute the flux of the electric field through a cylinder of radius R and height h (not including the top and bottom discs), parallel to the z-axis, between $z = 0$ and $z = h$.

   (b) Compute the flux of the electric field through a cylinder of radius R and height $2h$ (including the top and bottom discs), parallel to the z-axis, between $z = -h$ and $z = h$.

   Note: In Physics 51, you'll learn about Gauss's Law, which states that the flux of an electric field through a closed surface is proportional to the total charge contained inside that surface. Because part (b) involves a closed surface, your answer should be proportional to q and should not depend on R or h.