Research Area: Geomeric Modeling and Design

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I am also interested in applying my knowledge in pure mathematics to other disciplines, especially to computer aided design and engineering mechanics. This part of my work is presented in [CG1], [CG2], [CG3], [CG4], [DGG], [FGM], [GGo] and [WG]. In this part of my research, in addition to involving undergraduates at HMC, I have been collaborating with experts in Computer Aided Geometric Design (CAGD): Professor Rida Farouki (UC Davis), Professor Ronald Goldman (Rice University) and Professor Jean Gallier (UPenn). I have also collaborated with Professor Philip Cha of the engineering department here at Harvey Mudd College. We have published three papers. Two are in the *Journal of Sound and Vibration*, a third one is in the *International Journal of Computational and Numerical Analysis and Applications.*

My work with J. Gallier and D. DeCarlo gives fast algorithms for computing control points for polynomial curves and surfaces given by parametric definitions. We derive recurrence formulae and closed-form expressions. Our methods have low polynomial (time and space) complexity, as opposed to the naive methods which have exponential complexity. I have also developed efficient methods to get polar forms and polar values. Polar forms (or blossoms) are n-variate multiaffine symmetric polynomials which are intimately related to some of the central ideas in computer aided geometric design, namely B-spline curves and surfaces. With one of my students, we have sought minimal symmetrizing sets of multiaffine polynomials.

My work with R. Farouki and H. Moon on Minkowski roots of complex sets involves "factoring" a complicated geometric object into Minkowski products of certain simpler ones. These novel ideas for designing geometric objects will provide new mathematical foundations in computer aided geometric design and will help to design geometric objects more efficiently and precisely. This work was recognized by an invitation for a presentation at the conference of Theory and Applications in Geometric Modeling and Processing 2000 held in Hong Kong, April, 2000.

I have also collaborated with HMC student, Micah Smukler, and extended the work of Farouki and his collaborators on the Minkowski product of complex sets to the Minkowski product of quaternion sets. This work has important

applications for efficiently generating a wide range of surfaces, including most of the popular surfaces needed in industry, such as pipe surfaces, canal surfaces and ruled surfaces. A generalization of the quaternion Minkowski product reduces in special cases of Laguerre geometry. It has been recently discovered that this has great applications in CAGD. Most importantly, the techniques developed in this work can be used to design certain geometric objects such as geometric objects with certain singularities that can not be designed using current techniques in CAGD. This work was presented at the SIAM Conference on Geometric Design and Computing, November 5, 2001 in Sacramento, California. A paper on this research has been submitted to *Geometriae Dedicata*.

Finally, I have been involved in a wide range of undergraduate activities (with math and/or math/computer science majors) incorporating computer technology in teaching, learning and research. Besides supervising students obtaining mathematical theorems, I also supervise students using computer technology to help in conducting research by writing Maple programs to aid in calculations such as expanding the complicated PDEs derived in our research. I guide students to implement the mathematical results obtained to visualize the geometric objects, such as using Maple or Mathematica to draw level sets or projections for the associative cycles and Cayley cycles obtained in our research. I also write computer programs with students to evaluate a calibration (whose comass has been calculated) on the relevant Schubert cycles and output the Schubert indices on which the calibration achieves its comass. In addition, I have supervised several students, and have developed a multi-media course on "Applying the Geometry of Curves and Surfaces to Computer Aided Geometric Design."

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