$\frac{100}{100}$ EJ, NH

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Calc II (Math 61)
Section 2
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Homework 1: 4.1 # 5, 11, 14, 21(A)

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(5) Find the Taylor polynomial p_k of $f(x)=\sqrt{x}$, $k=3$ at point $a=9$.

$f(a)=\sqrt{9}=3$ $f'(x)=\frac{1}{2\sqrt{x}}$ $f'(a)=\frac{1}{2\sqrt{9}}=\frac{1}{6}$ $f''(a)=\frac{-1}{4\sqrt{x^3}}=\frac{-1}{4\sqrt{9^3}}=\frac{-1}{108}$ $f'''(a)=\frac{1}{3888}$

$\boxed{p_k(x)=3+\frac{1}{6}(x-9)-\frac{1}{216}(x-9)^2+\frac{1}{3888}(x-9)^3}$ ✓

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(11) Find the 1st + 2nd order Taylor polynomials for $f(x,y)=e^{2x}\cos 3y$ at $\vec{a}=(0,\pi)$

$f_x=2e^{2x}\cos 3y$ $f_y=-3e^{2x}\sin 3y$ $f_{xx}=4e^{2x}\cos 3y$ $f_{xy}=-6e^{2x}\sin 3y=f_{yx}$

$f_{yy}=-9e^{2x}\cos 3y$ $f_x(a)=-2$ $f_y(a)=0$ $f_{xx}(a)=-4$ $f_{xy}(a)=0$ $f_{yy}(a)=9$ $f(a)=-1$

$p_1(0,\pi)=-1-2x+0$ $\boxed{p_1(0,\pi)=-1-2x}$ ✓

$p_2(0,\pi)=-1-2x+\frac{1}{2}(-4)(x-0)^2+0+\frac{1}{2}(9)(y-\pi)^2$

$\boxed{p_2(0,\pi)=-1-2x-2x^2+\frac{9}{2}(y-\pi)^2}$ ✓

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(14) Calculate the Hessian matrix $Hf(\vec{a})$ for $f(x,y)=\frac{1}{x^2+y^2+1}$ $\vec{a}=(0,0)$

$f_x=\frac{-2x}{(x^2+y^2+1)^2}$ $f_y=\frac{-2y}{(x^2+y^2+1)^2}$ $f_{xx}=\frac{(x^2+y^2+1)^2(-2)+2x\,2(x^2+y^2+1)2x}{(x^2+y^2+1)^4}=\frac{-2(x^2+y^2+1)^2+6x^2(x^2+y^2+1)}{(x^2+y^2+1)^4}$

$f_{xy}=f_{yx}=\frac{-2x(x^2+y^2+1)\,2x}{(x^2+y^2+1)^4}=\frac{-4x^2(x^2+y^2+1)}{(x^2+y^2+1)^4}$ $Hf(0,0)=\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy}\end{bmatrix}=\boxed{\begin{bmatrix} -2 & 0 \\ 0 & -2\end{bmatrix}=Hf(0,0)}$ ✓

$f_{yy}=\frac{-2(x^2+y^2+1)^2-6y^2(x^2+y^2+1)}{(x^2+y^2+1)^2}$

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(21) $\boxed{A}$ Find the 3rd order polynomial of $f(x,y,z)=x^4+x^3y+2y^3-xz^2+x^2y+3xy-z+2$ at $(0,0,0)$.

xyz
yxz
yzx
zyx
zxy
xzy

$f_x=4x^3+3x^2y-z^2+2xy+3y$ $f_y=x^3+6y^2+x^2+3x$ $f_z=-2xz-1$

$f_{xy}=f_{yx}=3x^2+2x+3$ $f_{xz}=f_{zx}=-2z$ $f_{yz}=f_{zy}=0$ $f_{xx}=12x^2+6xy+2y$

$f_{yy}=12y$ $f_{zz}=-2x$ $f_{xxx}=24x+6y$ $f_{xxy}=6x+2$ $f_{xxz}=0$ $f_{yyy}=12$ $f_{yyx}=0$ $f_{yyz}=0$

$f_{zzz}=0$ $f_{zzx}=-2$ $f_{zzy}=0$ $f_{xyz}=0$

$p_3(x,y,z)=f+(f_x)(x)+(f_y)(y)+(f_z)(z)+\frac{1}{2}f_{xx}(x)^2+(f_{xy})(xy)+(f_{xz})(xz)+\frac{1}{2}(f_{yy})(y)^2+(f_{yz})(yz)+$
$\frac{1}{2}(f_{zz})(z)^2+\frac{1}{6}(f_{xxx})(x)^3+\frac{1}{6}(f_{yyy})(y)^3+\frac{1}{6}(f_{zzz})(z)^3+\frac{1}{2}(f_{xxy})(x^2y)+\frac{1}{2}(f_{xxz})(x^2z)+$
$\frac{1}{2}(f_{yyz})(y^2z)+\frac{1}{2}(f_{yyx})(y^2x)+\frac{1}{2}f_{zzx}(z^2x)+\frac{1}{2}(f_{zzy})(z^2y)+(f_{xyz})(xyz)$

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$p_3(0,0,0)=2+0+0-1z+0+3xy+0+0+0+0+0+2y^3+0+x^2y+0+0+0$
$-z^2x+0+0$

$\boxed{p_3(0,0,0)=2-z+3xy+2y^3+x^2y-z^2x}$ ✓