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Corrigendum

Corrections to "Oxtoby's pseudocompleteness revisited" [Topology and its Applications 100 (2000) 119–132] ☆

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We are indebted to Emil Jerabek of Charles University for pointing out to us in an e-mail in March 2000 an example showing that the first assertion in Theorem 3.10 is incorrect. (Suppose X is the unit interval $[0, 1]$, let $\mathcal{B}_n = \{[a, b]\colon\ a, b \in X,\ a < b\}$, for $n \in \omega$, and let $A = (0, 1]$. Although A is present for the Oxtoby sequence $(\mathcal{B}_n)$, the induced sequence $(\mathcal{B}_n|A)$ of π^0-bases fails to be an Oxtoby sequence for A since $((0, 1/2^n))_n$ is an associated nest with empty intersection.)

This impacts the remainder of the paper as follows:

(1) Theorem 3.10 relates to an open question of Aarts and Lutzer [1]: Is every dense G_δ-subspace of a pseudocomplete space pseudocomplete? As is noted in [2, 3.9J], this inheritance is well-known for Baire in place of pseudocomplete. The following replacement for Theorem 3.10 improves the inheritance of Baire at the cost of a stronger property for the superspace; it also performs, as will be seen, some of the duties assigned to the original.

Theorem 3.10′. *If A is a G_δ-subspace of an Oxtoby space (X, τ) and is present for some Oxtoby sequence $(\mathcal{B}_n)$ for X, then A is a Baire space.*

Proof. There is (U_n) in τ such that $A = \bigcap_n U_n$. Suppose that for each n, $V_n \in \tau$ is such that $V_n \cap A$ is dense in A, and let W be in τ with $W \cap A \neq \emptyset$. To establish that A is Baire, we need only show $\bigcap_n V_n \cap W \cap A \neq \emptyset$. Let $T_n = V_n \cap U_n \cap W$. Then each $T_n \subset U_n \cap W$ and each $T_n \cap A$ is dense in $W \cap A$.

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For the induction hypothesis, suppose $\mathcal{N} = (B_k)_{k<n}$ is an associated nest for $(\mathcal{B}_k)_{k<n}$ with $\operatorname{int}(\bigcap\mathcal{N}) \cap W \cap A \neq \emptyset$ and $B_k \subset T_k$ for each $k < n$.

From this hypothesis $W \cap A \supset T_n \cap \operatorname{int}(\bigcap\mathcal{N}) \cap A \neq \emptyset$. Since A is present for $\mathcal{B}_n$, there is $B_n \in \mathcal{B}_n$, such that $B_n \subset T_n \cap \operatorname{int}(\bigcap\mathcal{N}) \subset W$, and $\operatorname{int}(B_n) \cap A \neq \emptyset$. Now, by the above and $\operatorname{int}(B_n) = \operatorname{int}\bigcap(B_k)_{k<n+1}$, the induction hypothesis may be seen to be satisfied for the nest $(B_k)_{k<n+1}$. By mathematical induction, there is an associated nest (B_n) for $(\mathcal{B}_n)$ satisfying, for each n, $B_n \subset T_n$ (and $\operatorname{int}(B_n) \cap A \neq \emptyset$). As $(\mathcal{B}_n)$ is an Oxtoby sequence, $\bigcap_n B_n \neq \emptyset$. Moreover, $\bigcap_n V_n \cap W \cap A = \bigcap_n V_n \cap W \cap \bigcap_n U_n = \bigcap_n T_n \supset \bigcap_n B_n \neq \emptyset$. □

(2) Jerabek's example, modified slightly, applies to the sequence in Example 3.11, but it follows from Theorem 3.10′ that each G_δ-subspace of the Sorgenfrey line is a Baire space. Moreover, it can be shown directly that each such subspace is pseudocomplete.
(3) At present, Corollary 3.12 is unproved and is open for dense G_δ subspaces. If *pseudocomplete* is replaced by *Baire* in the conclusion, the result becomes a corollary of our revised Theorem 3.10′ above.
(4) Proposition 3.14 should be replaced by:

Proposition 3.14′. *A regular bitopological space, (X, τ, τ^*) is countably subcompact if and only if it has a basic Oxtoby sequence $(\mathcal{B}_n)$ of the form $\mathcal{B}_{2k} = \mathcal{B}$, $\mathcal{B}_{2k+1} = \{\mathrm{cl}^*(B)\colon B \in \mathcal{B}\}$ for some base $\mathcal{B}$. Thus each countably subcompact space is pseudocomplete.*

Proof. Suppose $\mathcal{B}$ is a base such that each strong nest in $\mathcal{B}$ has nonempty intersection. We may assume $\emptyset \notin \mathcal{B}$. Let $(\mathcal{B}_n)$ be of the form as in the proposition. For $x \in U \in \tau$, there is $B \in \mathcal{B}$ with $x \in B$ and $\mathrm{cl}^*(B) \subset U$. For $n < \omega$, either $B \in \mathcal{B}_n$ or $\mathrm{cl}^*(B) \in \mathcal{B}_n$. So, directly or by regularity, each $\mathcal{B}_n$ contains a neighborhood base at each x, thus is a π^0-base. We need only show now that $(\mathcal{B}_n)$ is an Oxtoby sequence. Let (C_n) be an associated nest for $(\mathcal{B}_n)$. There is $(B_n) \subset \mathcal{B}$ with $C_{2k} = B_{2k}$ and $C_{2k+1} = \mathrm{cl}^*(B_{2k+1})$. Now, as

$$C_{2k} = B_{2k} \supset C_{2k+1} = \mathrm{cl}^*(B_{2k+1}) \supset C_{2k+2} = B_{2k+2},$$

$(B_{2k}) \subset \mathcal{B}$ is a strong nest. Thus (as nests are descending) $\bigcap_n C_n = \bigcap_k C_{2k} = \bigcap_k B_{2k} \neq \emptyset$.

Suppose $(\mathcal{B}_n)$ is an Oxtoby sequence of the form indicated in the proposition. We need only show that each strong nest $(B_n) \subset \mathcal{B}$ has nonempty intersection. Defining $C_{2k} = B_{2k}$ and $C_{2k+1} = \mathrm{cl}^*(B_{2k+1})$, we have (C_n) an associated nest for $(\mathcal{B}_n)$. Thus $\bigcap_n B_n = \bigcap_k B_{2k} = \bigcap_k C_{2k} = \bigcap_n C_n \neq \emptyset$. □

(5) Corollary 3.15 and Example 3.16 are correct as they stand.

References

[1] J.M. Aarts, D.J. Lutzer, Pseudo-completeness and the product of Baire spaces, Pacific J. Math. 48 (1973) 1–10.
[2] R. Engelking, General Topology, Heldermann Verlag, Berlin, 1989.