Homework Assignment #11

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Instructions: This assignment is due at our meeting in class on Wednesday, December 10th. You are encouraged to work together on the problems but the final write-up that you submit must be done individually.

1. In your text, *Introduction to Stochastic Processes*, by Hoel, Port, and Stone, read Sections 6.1-6.3. Then solve the following exercises in Chapter 6:
   (a) Exercise 2
   (b) Exercise 11
   (c) Exercise 13
   (d) Exercise 15

2. In Exercise 20(d) of Chapter 4 you were asked to find the mean and covariance function of the process defined by

$$X(t) = W(t) - tW(1)$$

for $0 \leq t \leq 1$, where $W(t)$ was a Wiener process (Brownian motion). This process is usually referred to as a Brownian bridge or "tied down" Brownian motion (since $X(1) = 0$). In this problem we consider the Brownian bridge for the case in which the Wiener process is standard Brownian motion $\{B(t) : t \geq 0\}$, i.e. $\sigma^2 = 1$, and we denote the corresponding Brownian bridge by $\{B_0(t) : 0 \leq t \leq 1\}$

   (a) Show that the process $\{Y(t) = (t+1)B_0\left(\frac{t}{t+1}\right) : t \geq 0\}$ is standard Brownian motion.
   (b) For $0 < s < 1$, find the distribution of

$$M_0(s) = \max\{B_0(u)/(1-u) : 0 \leq u \leq s\}.$$

Hint: Use the result of the first part of this problem to reduce the question to finding the distribution of $M(t) = \max\{B(v) : 0 \leq v \leq t\}$ (which we know) for a particular t (which we can express in terms of s).