

Homework Assignment #4

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Instructions: This assignment is due in class on Wednesday, October 1st. You are encouraged to work together on the problems but the final write-up that you submit must be done individually.

1. In your text, *Introduction to Stochastic Processes*, by Hoel, Port, and Stone, read Sections 1.4-1.6 and solve the following exercises in Chapter 1:

- (a) Exercise 13
- (b) Exercise 14
- (c) Exercise 18

2. A typical assistant professor at University X is hired at one of six levels or “states” which we designate 1, 2, 3, 4, 5, 6. State 7 corresponds to tenure and state 8 corresponds to “leaving the university”. An assistant professor in state $x \leq 5$ may move to state $x + 1$ or to state 8. An assistant professor in state 6 may move to state 7 or state 8. A study has been done and data collected. On the basis of the data the tenure system at University X is modeled as a Markov chain with transition probability matrix

$$P = \begin{bmatrix} 0 & .5 & 0 & 0 & 0 & 0 & 0 & .5 \\ 0 & 0 & .6 & 0 & 0 & 0 & 0 & .4 \\ 0 & 0 & 0 & .5 & 0 & 0 & 0 & .5 \\ 0 & 0 & 0 & 0 & .5 & 0 & 0 & .5 \\ 0 & 0 & 0 & 0 & 0 & .8 & 0 & .2 \\ 0 & 0 & 0 & 0 & 0 & 0 & .01 & .99 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and initial probability vector $\pi_0 = [.9 \ 0 \ 0 \ .1 \ 0 \ 0 \ 0 \ 0]$.

- (a) What are the closed sets? What are the equivalence classes?
- (b) What is the probability that an assistant professor receives tenure?
- (c) Find $\rho_{1,8}^{(3)} = P_1(T_8 = 3)$, i.e. the probability that an assistant professor at level 1 is dismissed on his 3rd review.

3. The Media Police have identified six states associated with television watching: 0 (never watch TV), 1 (watch only PBS), 2 (watch TV fairly frequently), 3 (addict), 4 (undergoing behavior modification), 5 (brain dead). Transitions from state to state can be modeled as a Markov chain with the following transition probability matrix:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ .5 & 0 & .5 & 0 & 0 & 0 \\ .1 & 0 & .5 & .3 & 0 & .1 \\ 0 & 0 & 0 & .7 & .1 & .2 \\ 1/3 & 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

- (a) Which states are transient and which are recurrent?
- (b) Starting from state 1, what is the probability that state 5 is entered before state 0; i.e. what is the probability that a PBS viewer will end up brain dead?
- (c) Assuming transitions from state to state take place on a monthly basis, find the expected number of transitions, starting from state 1, until either state 5 or state 0 is entered; i.e. the expected amount of time for a PBS viewer to either give up TV entirely or end up brain dead.