

Homework Assignment #7

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Instructions: This assignment is due in class on Wednesday, October 29. You are encouraged to work together on the problems but the final write-up that you submit must be done individually.

1. In your text, *Introduction to Stochastic Processes*, by Hoel, Port, and Stone, read Section 3.1. Also, read carefully the handouts “Exponential Random Variables and Explosions” and “Verifying the Markov Property in Continuous Time”.
2. Harry dreams he is Prince Charming coming to rescue Sleeping Beauty (SB) from her slumbering imprisonment with a kiss. The situation is more complicated than in the original tale, however, as SB sleeps in one of three positions: (1) flat on her back, in which case she looks truly radiant; (2) fetal position, in which case she looks less than radiant; (3) fetal position and sucking her thumb, in which case she looks radiant only to an orthodontist. SB’s changes of position occur according to a Markov pure jump process $\{X(t) : t \geq 0\}$ with jump probability matrix

$$Q = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & .75 & .25 \\ .25 & 0 & .75 \\ .25 & .75 & 0 \end{bmatrix} \end{matrix}.$$

SB stays in each position for an exponential distributed amount of time with rate q_x , $x = 1, 2, 3$, measured per hour, where

$$q_1 = 1/2, \quad q_2 = 1/3, \quad q_3 = 1.$$

Assume that SB starts sleeping in the truly radiant position.

- (a) Find the infinitesimal generator matrix for this process.
- (b) Find the stationary distribution for this process.
- (c) Find the stationary distribution for the imbedded chain $\{X_n\}$.
- (d) How are these stationary distributions related?
- (e) Simulate 200 sample paths through the first 100 jumps for this process (including the time of each jump).

- (f) Graph 2 sample paths through the first 10 jumps.
- (g) Estimate the distribution of $X(t)$ at times $t = 20$ and $t = 50$ hours from your data (ignore paths which don't last long enough).