Homework Assignment #9

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Instructions: This assignment is due in class on Wednesday, November 12. You are encouraged to work together on the problems but the final write-up that you submit must be done individually.

1. Read carefullly the handouts "Joint Normal Random Variables" and "Construction of Brownian Motion". In your text, *Introduction to Stochastic Processes*, by Hoel, Port, and Stone, read Sections 4.1, 4.2, and 4.3. Then solve the following exercises in Chapter 4:

   (a) Exercise 6

   (b) Exercise 8 (using the joint characteristic function as defined in the handout "Joint Normal Random Variables" is one way to solve this problem)

   (c) Exercise 18

   (d) Exercise 19

   (e) Exercise 20b,d

   For the next two questions you can use the method described in the handout "Construction of Brownian Motion" to simulate the continuous path approximating processes $\{B^{(n)}(t) : 0 \leq t \leq 1\}$.

   (a) Using your favorite method, simulate 200 values of the sequence of *independent* random variables

   $$\left\{ V(1),\ V(\frac{2k+1}{2^{n+1}}) : k = 0, 1, \ldots, 2^n - 1,\ n = 0, 1, \ldots, 9 \right\}$$

   where $V(1)$ has a $N(0, 1)$ distribution and, for all $n \geq 0$, each $V(\frac{2k+1}{2^{n+1}})$ has a $N(0, \frac{1}{2^n})$ distribution. Thus, as examples, for $n = 0$ we have $V(\frac{1}{2})$ with a $N(0, 1)$ distribution, and for $n = 1$ we have $V(\frac{1}{4})$ and $V(\frac{3}{4})$ with $N(0, \frac{1}{2})$ distributions.

   (b) Using your 200 simulated values of

   $$\left\{ V(1),\ V(\frac{2k+1}{2^{n+1}}) : k = 0, 1, \ldots, 2^n - 1,\ n = 0, 1, \ldots, 9 \right\}$$

simulate 200 sample paths of $\{B^{(n)}(t) : 0 \leq t \leq 1\}$ for $n = 4$ and $n = 9$. For each of these values for n, graph two of the sample paths. For each of these values of n, show a histogram of the values of $B^{(n)}(1/2)$ and $B^{(n)}(1)$ that you obtained.