HMC Math 62- Probability & Statistics
QUIZ II: Due Friday, June 18, 2004 (75 minute quiz)

0: *(1 Point)* Put your name and section number on your quiz.

1: *(14 Points)* Suppose I model ships arriving at the Babylon 5 station as a Poisson process, and I see an average of 36 ships per day. Given this model, answer the following:

> *(i)*: What probability distribution models the *time* until the next ship arrives (give distribution name and parameter(s))? What is the probability that the first ship of the day appears within the first hour of the day?
>
> *(ii)*: Just like the Binomial, the Poisson random variable is well-approximated by a normal distribution with the same mean and variance. Given this fact, give an approximation, in terms of the cdf $\Phi(z)$ for N(0,1), for the probability that 40 or more ships arrive in one day.

2: *(7 Points)* Suppose $X \sim \text{Geo}(p)$. Prove that X is *memoryless*; that is, for positive integers s and t, show that $P(X = s + t \mid X > s) = P(X = t)$.

Hint: You *may* use $1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$ when $|x| < 1$, but there's an easier way to find $P(X > s)$... what, precisely, is happening when $\{X > s\}$?

3: *(28 Points)* Suppose continuous r.v. X has density function $f_X(x) = k(x - 1)\ ,\ \ 1 \leq x \leq 3$.

> *(i)*: Find the value of k for which this is a valid density function.
>
> *(ii)*: What are $E[X]$ and $\text{Var}(X)$?
>
> *(iii)*: Compute $F_X(x)$, the cumulative distribution function of X, and sketch its graph. Be sure to *fully* define $F_X(x)$.
>
> *(iv)*: Let $Y = 2X - 1$. Find the probability density function $f_Y(y)$ for Y.
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> **Hint:** First find the c.d.f. $F_Y(y)$ from the c.d.f. of X. Don't forget to note the range!