

# Homework 10 - KEY

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Note: Many problems can be solved in more than one way; we present only a single solution here.

## 1 Problem 10A

This problem is merely testing your understanding of null and alternate hypotheses, as well as the different types of error. In this case, our null hypothesis is  $H_0 : \sigma = .05$ , and our alternate hypothesis is:  $H_a : \sigma < .05$ . Notice that the null hypothesis is a strict equality; for the purposes of the hypothesis we're examining, it would make no difference if  $\sigma$  were larger than .05.

As usual, we have that a type I error is when we falsely reject the null hypothesis; the sheaths are purchased even though the standard deviation of their thickness is too large. Similarly, a type II error is when we falsely accept the null hypothesis; the sheaths are rejected even though the standard deviation of their thickness is small enough. Note that the type II error is not as dangerous; in general, a type I error is considered more serious than type II.

## 2 Problem 10B

For part a, notice that the alternate hypothesis does not specify whether  $p > 0.5$  or  $p < 0.5$ ; thus,  $R_1$  is most appropriate since it covers both possible reasons that we would reject the null hypothesis.

For part b, we have a type I error when  $p = 0.5$ , but we claim that it does not. We have a type II error when  $p \neq 0.5$ , but we claim that it does.

For part c, when  $H_0$  is true, we are counting the number of people who prefer company 1, with a 0.5 probability that any given person prefers company 1. It follows that  $X \sim \text{Bin}(25, 0.5)$ . To find the probability of a type I error, we then sum the probabilities that  $X$  is in the rejection region:

$$\sum_{n=1}^7 \binom{25}{n} (0.5)^{25} + \sum_{n=18}^{25} \binom{25}{n} (0.5)^{25} = \boxed{0.0433}$$

For part d, the probability of a type II error is simply the probability that  $X$  is not in the rejection region. For both  $p = 0.3$  and  $p = 0.7$ , this is:

$$\sum_{n=8}^{17} \binom{25}{n} (0.3)^n (0.7)^{25-n} = \boxed{0.4881}$$

In the case of  $p = 0.4$  and  $p = 0.6$ , the sum becomes:

$$\sum_{n=8}^{17} \binom{25}{n} (0.3)^n (0.7)^{25-n} = \boxed{0.8452}$$

Note that the probability of a type II error rises as the true value of  $p$  approaches the hypothesized value (in this case,  $p = 0.5$ ); the closer the two are together, the easier it is to mistake data from one for data from the other.

For part e, since the result of 6 falls within our rejection region, we would reject the null hypothesis and conclude that  $p \neq 0.5$ .

### 3 Problem 10C

For each of these problems, the significance level is simply the probability that a randomly selected instance of  $Z$  ends up in the rejection region, assuming  $H_0$  is true (that is, the probability of a type I error.) Since the distribution of  $Z$  is the standard normal distribution, we merely need to consult a cumulative standard normal distribution table to find that the answers are approximately  $\boxed{0.03}$ ,  $\boxed{0.003}$ , and  $\boxed{0.004}$  respectively.

### 4 Problem 10D

For part a, the parameter of interest is average braking distance at 40 mph under the specified conditions. Our null hypothesis is once again the one

that errs on the side of caution; thus,  $H_0$ : distance = 120 ft. Similarly, our alternate hypothesis is  $H_a$ : distance < 120 ft.

For part b, we only want to reject the null hypothesis if our average braking distance is significantly less than 120 feet. Thus, the most appropriate rejection region is  $R_2$ .

For part c, the significance level of  $R_2$  is the probability of a type I error. Assuming  $\mu = 120$ , this is:

$$P(\bar{x} \leq 115.20) = P\left(Z \leq \frac{115.20 - 120}{\frac{10}{\sqrt{36}}}\right) = P(Z \leq -2.88) = 0.00199$$

In order to change the region to reduce the probability of a type I error to 0.001, we merely solve the equation:

$$\phi\left(\frac{x - 120}{\frac{10}{\sqrt{36}}}\right) = 0.001$$

After a short bit of algebra, we find that the necessary rejection region is  $\{\bar{x}; \bar{x} \leq 114.85\}$ .

For part d, we are asked to find the probability of a type II error when the braking distance is 115 feet. This occurs when  $\bar{x}$  is not in the rejection region, so its probability is equal to:

$$P(115.20 \leq \bar{x}) = P\left(\frac{115.20 - 115}{\frac{10}{\sqrt{36}}} \leq Z\right) = P(0.12 \leq Z) = 0.45224$$

For part e, we simply repeat the methods used in problem 10C; this problem is merely asking the significance level for the appropriate regions when the distribution is transformed into a standard normal. The answers are 0.0099 and 0.00199 respectively.

## 5 Problem 10E

For part a, we first find the appropriate rejection region as dictated by the test's parameters. For a two-tailed level 0.01 test, we accept if the result is in the region:

$$\left(-Z_{\frac{\alpha}{2}} \leq \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq Z_{\frac{\alpha}{2}}\right)$$

With our given parameters and a little bit of algebra, we find that the accepting region is:

$$(94.226 \leq \bar{x} \leq 95.774)$$

Since the given value of  $\bar{x}$  falls within our accepting region, we accept the null hypothesis.

For part b, the probability of a type II error is the probability that the result of our test is in the accepting region; that is,

$$P(94.226 \leq \bar{x} \leq 95.774) = P\left(\frac{94.226 - 94}{0.3} \leq Z \leq \frac{95.774 - 94}{0.3}\right) = \boxed{0.2266}$$

For part c, we simply use the formula as listed in the book;  $Z_{\frac{\alpha}{2}}$  and  $Z_{\beta}$  are obtained from our desired probabilities of type I and type II errors.

$$n = \left[ \frac{\sigma(Z_{\frac{\alpha}{2}} + Z_{\beta})}{\mu' - \mu} \right]^2 = \left[ \frac{1.2(1.29 + 2.58)}{95 - 94} \right]^2 = 21.56$$

Since we cannot have a fractional sample, we round up to  $\boxed{22}$ .

## 6 Problem 11-3

Part a of this problem is solved in an identical manner to part a of problem 10E; we determine an appropriate accepting region, and see if our sample falls within it. The accepting region turns out to be:

$$(0.254938 \leq \bar{x} \leq 0.255062)$$

The random sample clearly does not fall in that range, so we reject the null hypothesis.

For part b, we once again use the formula listed in the book. This time, we have  $Z_{\frac{\alpha}{2}} = 1.96$  and  $Z_{\beta} = 1.29$ , which gives us:

$$n = \left[ \frac{\sigma(Z_{\frac{\alpha}{2}} + Z_{\beta})}{\mu' - \mu} \right]^2 = \left[ \frac{0.0001(1.29 + 1.96)}{0.255 - 0.2552} \right]^2 = 2.641$$

Again, we cannot have a fractional sample, so we must take  $\boxed{3}$  samples.