Math 164 - HW 11

HW 11. *Call for Presentations begins immediately.*

1. Consider the functional J defined by $J(u) = \int_0^1 \frac{1}{2}(u')^2 - \frac{1}{4}u^4 \, dx$, where the domain consists of 'suitably smooth' functions $u : (0,1) \to \mathbb{R}$.

   (More precisely, the domain is the Hilbert Sobolev space $H = H_0^{1,2}((0,1))$ of functions with compact support (zero at the endpoints), with one generalized derivative, living in L^2, the space of square-integrable functions.)

   (a) Compute (by hand waving at some convergence theorems where needed) $J'(u)(v) = \lim_{t\to 0} \frac{1}{t}(J(u+tv) - J(u))$.

   (b) Compute (by hand waving at some convergence theorems where needed) $J''(u)(v,w) = \lim_{t\to 0} \frac{1}{t}(J'(u+tw)(v) - J'(u)(v))$.

2. Consider the functional J defined by $J(u) = \int_\Omega \frac{1}{2}\nabla u \cdot \nabla u - \frac{1}{4}u^4 \, dx$, where the domain is $H_0^{1,2}(\Omega)$. Use Green's second identity to deduce that if u satisfies the PDE $\Delta u + u^3 = 0$ on Ω with the boundary condition $u = 0$ on $\partial\Omega$, then $\nabla J(u) = 0$.

   Again, some convergence results will be assumed, implicitly.