Math 132 — HW 5 — due Tuesday, October 3.

Do:
Chapter 7 (18, 19) and **Chapter 8** (1).

And also:

A.) Find a solution to the following differential equation on $[0,1]$ using the method below:

$$\frac{df}{dx} = xf(x), \qquad f(0) = 1. \qquad (*)$$

(a) Transform $(*)$ into an integral equation.

(b) Construct an operator on functions

$$T : C_b([0,1]) \to C_b([0,1])$$

that is a *contraction* on $C_b([0,1])$ (verify this) and whose fixed point satisfies $(*)$.

(Recall that the metric on C_b is $d(f,g) = \|f-g\| = \sup_{[0,1]} |f(x) - g(x)|$.)

(c) Use T(f) and the method of successive approximations to solve $(*)$ in terms of an infinite power series.

(d) What is the radius of convergence of your power series? Is your solution valid on more of $\mathbf{R}$ than just $[0,1]$? Why?

B.) (a) Show that the radius of convergence R of the power series $\sum c_n x^n$ is given by $\lim_{n\to\infty}(|c_n|/|c_{n+1}|)$ provided this limit exists.

(b) Give an example of a power series for which the limit does *not* exist.

C.) What is the radius of convergence of $\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$?

D.) In a few sentences, describe the main ideas in the course so far. What concepts would you test if you were to, say, write a midterm for this course?