

# Discovering Euler's rule for polytopes

Pallavi Jayawant (Bates College), Dan Schaal (South Dakota State University), Jim Seibert (Regis University), Jessica Sidman (Mount Holyoke College), Kevin Woods (University of California, Berkeley)

## Abstract

Students will discover Euler's rule for polytopes through guided discovery learning in this elementary school outreach activity.

## Introduction and Goals

The activity leader will come equipped with several models of three-dimensional polytopes. Once students are comfortable identifying the vertices, edges, and faces of a polytope, the leader should direct them to count the number of each for various polytope models on hand. The goal will be for the students to notice that the rule  $\# \text{ vertices} + \# \text{ faces} - \# \text{ edges} = 2$ .

## Materials

1. Polytope models (These can be made from "zaks", "polydrons" or other supplies.)
  - (a) Tetrahedron
  - (b) Square pyramid
  - (c) Triangular prism
  - (d) Cube
  - (e) Octahedron
  - (f) Dodecahedron (advanced classes)
  - (g) Icosahedron (advanced classes)
2. Small stickers for labelling during counting.

## Implementation

### Whole class activity

Using a specific model, the leader should introduce the concepts of vertex, edge, and face of a polytope. (Explain that a vertex is like the corner of a box and that a face is like the side of a box.) Use at least one other model to make sure that students understand the definitions. (Point to the various parts of polytope models and make sure that students can classify them correctly.)

## Group work




Divide the class into groups. Each group will receive a tetrahedron and will be instructed that their assignment is to count either vertices, edges, or faces. If the group is counting the vertices (respectively edges, faces), they should be instructed to place a numbered sticker on each one so they can keep track of their count more easily. If a group finishes early, they may count the other features of the polytope.

## Whole class activity

When all groups have finished their initial assignment, the class should come together to record the data that has been collected in a table containing a column for each feature. If there are disagreements on counts, be sure to resolve them at this time.




## Group work

Have groups count the features of the square pyramid and triangular prism. Your table may look like the one below.

	V	F	E
	4	4	6
	5	5	8
	6	5	9

## Whole class activity

Ask the students to look for a pattern. The leader may want to add a fourth column (vertices + faces) and/or a fifth (vertices + faces - edges) if a pattern is not noticed. Your table may look like the one below.

	V	F	E	V+F	V+F-E
	4	4	6	8	2
	5	5	8	10	2
	6	5	9	11	2

Now the leader tells the students that a cube has 8 vertices and 6 faces. Ask the students to guess the number of edges on a cube. Distribute cubes and have the students verify the counts. This exercise may be repeated for the octahedron and the more complicated shapes for more mature classes. You may want to end by asking the students if they expect the pattern they have observed to hold for all 3-dimensional polytopes. You may wish to emphasize that although they have gathered a lot of convincing evidence, something more has to be done so that we can know for sure the pattern continues.