

Tilings, a hands-on outreach presentation

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Abstract

This is an outline for a hands-on high school outreach talk of 50 minutes on tilings. By reducing the number of hands-on activities from 3 to 2, the talk could easily work at the pre-high school level.

We have modified a lecture given by Richard Stanley in July 2004. While Stanley's lecture is designed for a general audience, some of the material is at the post-secondary level and the amount of material included leaves no time for hands on activities.

We drop some of the advanced material in Stanley's lecture but make an effort to retain visual parts. We use the extra time to expand 3 topics of Stanley's lecture with hands-on activities.

Introduction and Goals

In July 2004, Richard Stanley gave a public lecture, Tilings, at the IAS-PCMI. The slides of the lecture are available at <http://www-math.mit.edu/~rstan/trans.html>. In what follows, references to slides refer to those of Stanley's lecture.

On slide 3, there is a list of 10 questions regarding tilings. We mainly address the first 2: "Is there a tiling?" and "How many?" Towards the end of our talk, we briefly look at a third question: "What is a typical tiling?"

Outline of Talk

0:00–0:10

(Slides 3–5) Begin by handing each person in the audience a kit consisting of a 6×10 board with one of several sets of pentominoes that can be used to tile the board. Two examples of such tilings appear on slide 5. (Use different colored paper for each kind of kit, especially if you plan to reuse the kits.) After giving the students 5 minutes to complete the puzzles, present slide 4 which is a complete list of pentominoes and ask the students what questions we could ask regarding tiling with pentominoes and what methods could we use to help answer their questions.

0:10–0:25

(Slides 6–13) Hand out a kit consisting of a 4×4 uncolored grid missing 2 opposite corners and a set of dimers and ask the audience to tile it. After a few students become suspicious, hint that it is not possible. Suggest that they color the grid like a chessboard and find a counting argument based on color. Hand out a second 4×4 grid missing 2 squares which if colored like a chessboard would be of different colors. Ask if they can come up with a conjecture based on these two examples on whether a square grid that is missing two squares can be tiled. Ask them to come up with a proof for the positive situation and finish by presenting slides 8–13.

0:25–0:40

(Slides 18–20) Introduce the Aztec diamonds and ask how many tilings of each are there. Pass out graph paper and ask students to list the tilings for orders 1, 2 and 3. Come to a consensus as to how many there are and speculate on a possible formula for their numbers. Finish by presenting slide 20 and point out how recent was the result.

0:40–0:45

(Slides 38–40 or 42) Introduce the question “What does a typical tiling look like?” by contrasting the typical tiling of an even sized square with dimers and that of the Aztec diamond. Present the Arctic Circle theorem and point out how recent the result was, demonstrating again that the area is an active field of investigation.

0:45–0:50

(Slides 52–57) Finish with slides of infinite tilings.