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<u>Topological Helly's Thms</u> $\mathcal{A}$ fin. family of closed hom. cells in $\mathbb{R}^d$ (S^d resp). If the intersection of every $d+1$ ($d+2$ resp) or fewer members of $\mathcal{A}$ is a hom. cell, then $\cap\mathcal{A}$ is a hom. cell.

<u>Q</u>: What about torus? Arbitrary manifold?

<u>Homotopical Helly's Thm</u> (Nikonorov, 1994)
$\mathcal{A}$ is a family of k open sets $A_1, \dots A_k$ in a top. space T s.t. $\pi_{k-2}(\cup A_i) = 0$. If the intersection of $k-1$ or fewer of A_i is homot. trivial then $\cap A_i \neq \emptyset$.

<u>Q</u>: What does a homological Helly's Thm look like?

. . . .

<u>KKM Lemma</u> $C_1 \dots C_{d+1}$ are closed sets covering Δ^d s.t. $\forall x \in \Delta^d$, $x \in C_i$ for some $i \in \operatorname{carr} x$. Then $\cap C_i \neq \emptyset$.

<u>Fact</u>: If we have S^d instead of Δ^d the theorem does not hold.

<u>Q</u>: What condition would guarantee nonempty intersection?

<u>Fact</u>: BFPT is equivalent to KKML.

<u>Q</u>: What is equivalent to Lefschetz FPT? (if $L(f) \neq 0$ then f has a fixed point).