Complex Dynamics
Math 189A
Spring 2001

Assignment # 1.
Due Tuesday, 20 March.

Reading: Chapter 1 of Beardon, at least sections 1.1, 1.5, and 1.6.

Recall that on problems marked (G) for 'group' you may cooperate, while on problems marked (I) for 'individual' the only help you may get is from me.

Problems from Beardon:

Section 1.5, page 14 #1, 2 (both I). For these problems you may use information gleaned from the picture of the Julia set J of $P(z) = z^2 - 1$ in Figure 1.5.1, p.13 of Beardon. F_0 and F_{-1} are the components of the Fatou set which contain 0 and -1 respectively. Q2 has a typo; the point in the first line should be $-\sqrt{(1+\sqrt{5})/2}$.

Section 1.6, page 18 #1 (G).

Additional problems:

1. (G) Show that the rational map $z \mapsto z^2$ has no indifferent fixed points, that $z \mapsto z^{-2}$ has no attracting fixed points, and that $z \mapsto z + z^2$ has no repelling fixed points.
2. (I) Given $P(z) = z^2 + c$, prove directly that there is an A such that $P^n(z) \to \infty$ if $|z| > A$. Hence $J(P)$ is bounded. (This is a general fact for rational $R(z)$, unless $J(R) = \overline{\mathbb{C}}$.)
3. (G) (a) Find the fixed points and the periodic points of period 2 of $P(z) = z^2 + i$. Determine their character. (Hint: You can find some roots of the quartic by 'guess and check'.
   (b) Try to do the same for $Q(z) = z^2 - z$. What goes wrong?
4. (I) (a) Show that every quadratic polynomial is conjugate to one and only one polynomial of the form $z \mapsto z^2 + c$. (To simplify this question, you need only consider conjugation by maps of the form $z \mapsto az + b$, not by general Möbius transformations.) Show also that $z^2 - z$ is conjugate to $z^2 - 3/4$. What can you conclude, from this and Question 3(b), about the periodic points of $z^2 - 3/4$?
   (b) Let $P(z) = z^2 + c$. Explain why $P(z) - z$ divides $P^2(z) - z$, and using this, show that if P has no periodic points of period 2, then $P(z) = z^2 - 3/4$.

Fact: The backward iterates $R^{-n}(z)$ of any $z \in J(R)$ are dense in $J(R)$, if $R(z)$ is rational.
Fact: The repelling periodic points are dense in $J(R)$, if $R(z)$ is rational.
Fact: For a polynomial $P(z)$, the Julia set is the boundary of the basin of attraction of ∞.

5. (G) Using some or all of the above facts, or otherwise, write a preliminary outline of a scheme to produce computer pictures of the Julia sets of rational functions, or just of polynomials. There are many possible approaches. Write less than a page; point form is fine. Try to identify the main issues your program would have to deal with.