

Assignment # 5.

Due Wednesday, 18 April, in class.

Reading: Sections 3.1 (The Fatou and Julia Sets) and 3.3 (Normal Families and Equicontinuity) of Beardon.

Recall that on problems marked (**G**) for ‘group’ you may cooperate, while on problems marked (**I**) for ‘individual’ you may not cooperate and the only help you may get is from me.

Problems from Beardon:

Section 3.1, page 51 #2 (**G**), 3 (**I**), 4 (**I**).

Section 3.3, page 59 #2 (**G**).

Additional problem:

- A. Continue preparation of your presentation. Make an appointment to meet with me to discuss it.

Hint for Q2, p51. To prove that D is in the Fatou set, use the definition of equicontinuity. An $\varepsilon/2$ argument works for all sufficiently large n . To prove that every attracting fixed point p lies in the Fatou set, show that the iterates converge uniformly on some neighbourhood of p as follows. Show that p has a neighbourhood on which $R(z)$ is a *contraction*: there’s some λ strictly less than 1 such that for all z in the neighbourhood,

$$|R(z) - R(p)| \leq \lambda|z - p|.$$

To do this, start by showing that on the disc of convergence of the Taylor series for $R(z)$ about p , $R(z)$ can be written as

$$R(z) = R(p) + R'(z)(z - p) + (z - p)^2 h(z),$$

where $h(z)$ is analytic and, on a closed subdisc of the disc of convergence, $h(z)$ is bounded.

A similar approach is useful for Q3, P51.