In this tutorial, we review the differentiation of trigonometric, logarithmic, and exponential functions.

### Trigonometric Functions

The derivatives of the basic trigonometric functions are given here for reference.

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin x$</td>
<td>$\cos x$</td>
</tr>
<tr>
<td>$\cos x$</td>
<td>$-\sin x$</td>
</tr>
<tr>
<td>$\tan x$</td>
<td>$\sec^2 x$</td>
</tr>
<tr>
<td>$\sec x$</td>
<td>$\sec x \tan x$</td>
</tr>
<tr>
<td>$\csc x$</td>
<td>$-\csc x \cot x$</td>
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The derivatives of $\sin x$ and $\cos x$ can be derived using the limit definition of the derivative. For $\sin x$,

\[
\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x + h) - \sin(x)}{h} = \lim_{h \to 0} \frac{(\sin x \cos h + \cos x \sin h) - \sin x}{h}
\]

\[
= \lim_{h \to 0} \left[ \sin x \frac{\cos h - 1}{h} + \cos x \frac{\sin h}{h} \right]
\]

\[
= \sin x \lim_{h \to 0} \left[ \frac{\cos h - 1}{h} \right] + \cos x \lim_{h \to 0} \left[ \frac{\sin h}{h} \right]
\]

\[
= \sin x(0) + \cos x(1)
\]

\[
= \cos x.
\]

The derivative of $\cos x$ is derived analogously. Then the remaining derivatives can be derived using the quotient rule, since all the other trigonometric functions are quotients involving $\sin x$ and $\cos x$.

### Example

The derivative of $\tan(x^2)$ is $\sec^2(x^2) \cdot \frac{d}{dx}(x^2) = 2x \sec^2(x^2)$ by the chain rule.
Logarithmic Functions

By the definition of the natural logarithm, \( \frac{d}{dx} \ln x = \frac{1}{x} \) for \( x > 0 \). Also, \( \frac{d}{dx} \ln |x| = \frac{1}{x} \) for all \( x \neq 0 \). To see this, suppose \( x < 0 \). Then \( \ln |x| = \ln(-x) \).

So

\[
\frac{d}{dx} \ln |x| = \frac{d}{dx} \ln(-x) = \frac{d}{dx} (-x) \left( \frac{1}{-x} \right) = (-1) \left( \frac{1}{-x} \right) = \frac{1}{x}.
\]

Example

By the chain rule, the derivative of \( \ln(x^3 + 5) \) is \( \frac{d}{dx} (x^3 + 5) \cdot \frac{1}{x^3 + 5} = \frac{3x^2}{x^3 + 5} \).

Exponential Functions

There is an elegant way to show that \( \frac{d}{dx} [e^x] = e^x \). We start with the identity \( \ln(e^x) = x \).

Differentiating both sides,

\[
\frac{d}{dx} [\ln(e^x)] = \frac{d}{dx} (x)
\]

\[
\frac{d}{dx} [\ln(e^x)] = 1
\]

\[
\frac{d}{dx} (e^x) \cdot \frac{1}{e^x} = 1
\]

\[
\frac{d}{dx} (e^x) = e^x.
\]

Since \( e^x \) is never 0, this derivation holds for all \( x \).

Example

The derivative of \( e^{-3x^2 + 2} \) is \( e^{-3x^2 + 2} \cdot \frac{d}{dx} (-3x + 2) = -3e^{-3x^2 + 2} \).
### Key Concepts

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[I’m ready to take the quiz.]  [I need to review more.]
[Take me back to the Tutorial Page]