Bernoulli Equation: \( \frac{dy}{dt} + p(t)y = q(t)y^b \)

Let \( z = y^{1-b} \). Then \( y = z^\frac{1}{1-b} \), so by the Chain Rule,

\[
\frac{dy}{dt} = \frac{1}{1-b} z^{\frac{1}{1-b} - 1} \frac{dz}{dt} = \frac{1}{1-b} z^{\frac{b}{1-b}} \frac{dz}{dt}.
\]

Substituting into the original equation,

\[
\frac{1}{1-b} z^{\frac{b}{1-b}} \frac{dz}{dt} + p(t)z^{\frac{1}{1-b}} = q(t)z^{\frac{b}{1-b}}
\]

\[
\frac{1}{1-b} \frac{dz}{dt} + p(t)z = q(t)
\]

Dividing through by \( z^{\frac{b}{1-b}} \),

\[
\frac{dz}{dt} + (1-b)p(t)z = (1-b)q(t).
\]

After solving this linear equation, use \( z = y^{1-b} \) to return to the original variables.