Harvey Mudd College Math Tutorial:

Product Rule for Derivatives

In Calculus and its applications we often encounter functions that are expressed as the product of two other functions, like the following examples:

- $h(x) = xe^x = (x)(e^x)$,
- $h(x) = x^2 \sin x = (x^2)(\sin x)$,
- $h(x) = e^{-x^2} \cos 2x = (e^{-x^2})(\cos 2x)$.

In each of these examples, the values of the function h can be written in the form

$$h(x) = f(x)g(x)$$

for functions $f(x)$ and $g(x)$. If we know the derivative of $f(x)$ and $g(x)$, the **Product Rule** provides a formula for the derivative of $h(x) = f(x)g(x)$:

$$\boxed{h'(x) = [f(x)g(x)]' = f'(x)g(x) + f(x)g'(x).}$$

Proof

We illustrate this rule with the following examples.

- If $h(x) = xe^x$ then

$$\begin{aligned} h'(x) &= (x)'e^x + x(e^x)' \\ &= e^x + xe^x. \end{aligned}$$

- If $h(x) = x^2 \sin x$ then

$$\begin{aligned} h'(x) &= (x^2)' \sin x + (x^2)(\sin x)' \\ &= 2x \sin x + x^2 \cos x. \end{aligned}$$

- If $h(x) = e^{-x^2} \cos 2x$ then

$$\begin{aligned} h'(x) &= (e^{-x^2})' \cos 2x + e^{-x^2}(\cos 2x)' \\ &= -2xe^{-x^2} \cos 2x - 2e^{-x^2} \sin 2x. \end{aligned}$$

Key Concepts

Product Rule

Let $f(x)$ and $g(x)$ be differentiable at x. Then $h(x) = f(x)g(x)$ is differentiable at x and $h'(x) = f'(x)g(x) + f(x)g'(x)$.

[I'm ready to take the quiz.] [I need to review more.]
[Take me back to the Tutorial Page]