Connections between the Dot Product and Cross Product

• Lagrange’s Identity

\[ \|u \times v\|^2 = \|u\|^2 \|v\|^2 - (u \cdot v)^2. \]

• Volume of a Parallelepiped

Consider the parallelepiped with adjacent sides \(a, b,\) and \(c.\)

The area of the base is \(\|b \times c\|\).

The height of the parallelepiped is \(\|a\| \cos \theta.\)

Thus, the parallelepiped has volume

\[ V = \|b \times c\| \|a\| \cos \theta. \]

But recall that \(u \cdot v = \|u\| \|v\| \cos \theta \) for \(u \neq 0, \ v \neq 0.\)

Thus,

\[ V = a \cdot (b \times c). \]

It doesn’t matter which two vectors define the “base” of the parallelepiped:

\[ a \cdot (b \times c) = c \cdot (a \times b) = b \cdot (c \times a). \]