

...the secret to having students succeed on the Putnam Exam is the three Cs: classify, conquer, and complete.

Putnam, Pizza & Problem Solving

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Ok, here's a difficult question for you...how can you get roughly 10% of the student body at your college to get up early on a Saturday and spend six hours working on an incredibly difficult exam for which many will get a score of zero?

While we don't have a complete answer, we have seen this happen each of the past several years at Harvey Mudd College with the nationwide contest called the William Lowell Putnam Mathematical Competition, that mother-of-all-math-exams which makes all other math exams seem trivial. The "Putnam Exam," as it is known, derives its character from the unique blend of problem-solving skills that it requires, as well as easy-to-state problems that are actually quite doable if you look at them in just the right way. For the past six years we have coached a Tuesday evening problem-solving seminar that prepares students to take the Putnam Exam each December, and we can tell you a few of the tricks that we've learned along the way to keep our seminar fun and engaging.

The biggest secret appears to be pizza. Of course, this is true for many student events—free food is always a big hit. However, for our problem-solving seminar, it sends the message that this is *not* a regular class but a chance to have some fun. About fifty to seventy people show up each Tuesday.

A second consideration is format; we try to mix things up a bit for the hour and a half session. First, one of us pres-

ents a five-minute introduction on the *subject du jour*—whether it be "Peculiar Properties of Polynomials" or "Deduction by Induction." Then we hand out a problem set on the subject, with four or five problems carefully chosen so that everyone can find something challenging, but doable. Usually one of the problems is a Putnam problem. Moreover, we always throw a problem in that is completely different and fun, such as showing that Passion does indeed have a square root, by solving the cryptarithm

$$\sqrt{PASSION} = KISS.$$

We encourage the students to work in small groups (and, in fact, more than one romance has bloomed in the seminar). Pizza arrives about half-way through, catered by the campus dining hall; we make an arrangement with them to trade student meal card credit for this pizza, so students have the option to have dinner at our seminar instead of the dining hall. After a short break to get a plate of pizza, students are then encouraged to present their solutions (while others chow down).

The key to the presentations is making students feel that getting up before a room full of their peers and presenting a solution is both a positive and fun experience, even if their solution leaves some room for improvement. We, as coaches, play the role of talk-show hosts: listening intently but keeping things light, and asking questions that the audience is afraid to ask, to ensure that everyone understands what the presenter is trying

to say. Sometimes we suggest alternative approaches, but we are always affirming speakers in what they have done and what great insights their solutions may suggest.

A few years ago, we realized that for some of our more mathematically mature students, our problem-solving seminar wasn't sufficiently challenging. To address this we split the seminar into two parts: "Putnam Classic" and "Putnam TNG" (The Next Generation—a reference for the trekkies among us). The problem sets we developed for Putnam TNG consist entirely of former Putnam problems, but each week is focused on a particular subject or problem-solv-



Natalie Hudson presents a problem to pizza-packing Putnam people.

ing technique. We require a little bit of outside work for Putnam TNG; to attend, students must have worked at least one of these problems in the week before (which they should be ready to present to the group). Students move back and forth between the two seminars that run simultaneously, depending on whether or not they have had time to attack that week's problem set.

As for the actual competition, we believe the secret to having students succeed on the Putnam Exam is the three Cs: classify, conquer and complete. First, students should look at the problems and try to classify them by the methods needed to attack them—deducing that a problem is susceptible to the Pigeonhole Principle or Strong Induction is often half the battle. Once a student has figured out which problem she wants to attack, it is time to conquer it—

that is, scratch out the solution and convince oneself that all the loose ends have been nailed down. Finally, and perhaps most importantly, she needs to complete the problem; write out a carefully worded, neatly written and well-organized solution. We tell students it is better to nail down a single problem completely than to turn in multiple half-baked solutions. (For confirmation of this recommendation see page 5.)

On the day of the exam, we provide pastries and orange juice for students as they sit down for the exam. Lunch time at the dining hall is a free-for-all discussion of who got what problems. By the end of the day, everyone is exhausted, but we can usually coax students into writing up some of the solutions for our Putnam bulletin board. Students like to visit this board throughout the year, either to re-live their Putnam experience

or to wistfully ponder whether they will take it next year!

Finally, we like to give students a souvenir of their grueling time together. To commemorate a particular exam, we create mugs emblazoned with a graphic associated with one of that year's Putnam problems, and give them to all students who have competed that year. Students like to collect each year's edition, and it gives them a nice reminder of a Saturday spent challenging themselves with twelve truly vexing math problems.

Our seminar accomplishes several goals. It gives students a chance to survey a variety of mathematical topics, learn problem-solving skills, and practice mathematical writing and presentation in an enjoyable social setting. We like to think that the fun of doing math in this way is the big draw, but of course, it could be the pizza... ■