Modeling coexistence in variable environments

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Math 197: Senior Thesis

Population Models

The Lotka-Volterra equation models species \( u, v \) as

\[
\begin{align*}
\dot{u} &= u f(u, v) \\
\dot{v} &= v g(u, v).
\end{align*}
\]

(1)

(2)

Here, \( f \) and \( g \) (growth functions) are defined as

\[
\begin{align*}
f(u, v) &= r_u (1 - u/K) \\
g(u, v) &= r_v v.
\end{align*}
\]

(3)

(4)

where \( r_u \) is the maximum per-capita growth rate, \( a_u \) is the competition coefficient, and \( K \) (the carrying capacity) is the maximum population level. This model ignores spatial variations, and as such a better model is

\[
\begin{align*}
\dot{u}(x, t) &= d_u u(x, t) + u f(x, u, v) \\
\dot{v}(x, t) &= d_v v(x, t) + v g(x, u, v)
\end{align*}
\]

(5)

(6)

where \( d_u \) (the diffusion coefficient is proportional to how rapidly a species moves in response to gradients, and \( K \) varies spatially.

When one considers spatial and temporal variation, Equations (3) and (4) are no longer equivalent. We consider population densities on a habitat \( \mathcal{L} \), and assume that \( u \geq 0 \) and \( K \geq 0 \), we find

\[
\begin{align*}
\int_0^\infty u(x, t) dx = \lim_{t \to 0} \int_0^\infty u(x, t) dx.
\end{align*}
\]

The growth model we chose did not make a difference under spatial variation (Figure 2). However, when variation was high, Equation (3) allowed almost no coexistence, while Equation (4) seemed unaffected (Figure 3).

Population Health

Before each disturbance, the total population \( U \) was determined. A population was non-viable if:

1) \( U < 0.01 \) for five consecutive measurements.
2) \( U \) averaged < 0.025 over the final 10 measurements.
3) \( U \) exhibited near monotonically decreasing trend.

Coexistence was described as mutual viability.

Simulation Results

We found that as disturbance conditions change, there can be a reversal in which species is dominant (Figure 1).

Our Approach

In order to better understand the effect of temporal variability, we created a population model with periodic disasters. The environment was broken into 10 evenly spaced sections. \( N \) sections are randomly chosen and destroyed during each disaster. After 20 disasters, the population health is assessed.

Model Differences

In environments with spatially varying carrying capacities \( K = K(x) \), Equations (3) and (4) predict different behavior. If we consider the asymptotically stable population \( U^* = \lim u(x, t) \), and assume that \( u \geq 0 \) and \( K \geq 0 \), we find

\[
\begin{align*}
\int_0^\infty u(x, t) dx = \lim_{t \to 0} \int_0^\infty u(x, t) dx.
\end{align*}
\]

Proposition 1. As \( r/d \to 0 \), \( U^* \to \int_0^\infty K(x) dx \), regardless of whether we use Equation (3) or (4).

Proposition 2. As \( r/d \to 0 \), Equation (3) predicts \( U^* \to \int_0^\infty K(x) dx \), while Equation (4) predicts \( U^* \to \int_0^\infty K(x) dx \).

Conclusions

Our model shows that in a temporally varying environment, species parameters other that \( K \) may affect coexistence. When species pursue different competitive strategies. Indeed, this is much closer to what is observed in nature, which leads us to believe that such effects cannot be ignored.

It seemed that in environments with strong environmental gradients, coexistence was not possible if Equation (3) is used. Otherwise, the limiting factor to coexistence seems to be how long it takes for a stronger competitor to establish itself in a new environment.

Acknowledgments

I would like to thank Professor Adolph and Jeff Hellrung for help early on. Thanks to Professors Yong and Fowler for technical help. And thanks to Professor Jon Jacobsen, for inspiration, guidance, and moral support.

For More Information

Full thesis available at http://www.math.hamc.edu/~sstump/thesis/. For further questions, I can be contacted at ssstump@hmc.edu.