**Introduction**

Viscoelastic fluids are a broad class of fluids that exhibit both viscous and elastic properties. Fluid elasticity is the measure of a fluid’s tendency to return to its original shape in the absence of external forces, and fluid viscosity is the measure of its resistance to flow. Common examples of viscoelastic fluids include bio fluids, gels, egg white, and corn starch in water. Unlike Newtonian fluids, which have a constant viscosity, viscoelastic fluids have a viscosity that depends on the amount of stress being applied to the fluid. Our model is a generalization of incompressible fluid flow in one dimension in that it accounts for variations in fluid stress:

\[ u_t + uu_x = \sigma, \]

where \( u \) denotes fluid velocity and \( \sigma \) denotes fluid stress. To derive a governing equation for stress, we assume that each fluid particle can be approximated as a Maxwell element — a spring and damper connected in series. The variable \( t \) is used to denote strain.

We study a non-dimensionalized version of the (1–2) system with asymptotically constant boundary conditions. That is,

\[ u_t + uu_x = \sigma, \quad \sigma + uu_x = (\sigma + A)u_x - \sigma, \]

subject to the same boundary conditions as before. As \( \varepsilon \to 0 \), the solutions of the viscous PDE system look similar to those of the original. Traveling wave solutions to the viscous system must satisfy

\[ \sigma' = S' = (S + A)(L^2 - 25 - 1 - 2\varepsilon S) \]

**Regularization**

To better understand the behavior of the PDE system, when \( A \in (0, 1) \), we add a viscous term. This new term prevents the formation of shocks. The viscous PDE system is

\[ u_t + uu_x = \sigma + \varepsilon u_x, \quad \sigma + uu_x = (\sigma + A)u_x - \sigma, \]

subject to the same boundary conditions as before. As \( \varepsilon \to 0 \), the solutions of the viscous PDE system look similar to those of the original. Traveling wave solutions to the viscous system must satisfy

\[ \frac{\sigma'}{\sigma} = \frac{S'}{S} = \frac{(S + A)(L^2 - 25 - 1) - 2\varepsilon S}{2U} \]

**Computer Simulations**

To assist with the analysis of our PDE model, we developed a graphical user interface program, called VISCO, that simulates solutions. The numerical algorithm used by VISCO is a fractional-step Lax-Wendroff method [1]. We approximate a small step forward in time using a Taylor approximation.

\[ u(x, t_0 + \Delta t) = u(x, t_0) + \Delta u(x, t_0) = \frac{\Delta t^2}{2!} u_{xx}(x, t_0) + O(\Delta t^3) \]

\[ \sigma(x, t_0 + \Delta t) = \sigma(x, t_0) + \Delta \sigma(x, t_0) = \frac{\Delta t^2}{2} \sigma_{xx}(x, t_0) + O(\Delta t^3) \]

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**References**


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