Mathematics
Harvey Mudd College

Extending List Colorings of Planar Graphs
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Background

Let \( G \) be a graph with vertices \( \{v_1, \ldots, v_n\} \). A list assignment is a family of sets, \( \Phi = \{\Phi(v_i) : 1 \leq i \leq n\} \), where each \( \Phi(v_i) \) is a set of positive integers. A list coloring is a map \( \varphi : V(G) \rightarrow \bigcup_i \Phi(v_i) \) so that \( \varphi(v_i) \in \Phi(v_i) \) for all \( v_i \in V(G) \). A proper list coloring is one with \( \varphi(v_i) \neq \varphi(v_j) \) for \( v_i, v_j \in E(G) \). A graph \( G \) is \( k \)-choosable if there is a proper list coloring of \( G \) for every list assignment where \( |\Phi(v_i)| = k \) for all \( i \). The minimum \( k \) for which this condition holds is called the list colorability or choosability, denoted \( \chi_l(G) \).

Small Face

Theorem 3. (Böhme, Mohar and Stiebitz) Let \( G \) be a plane graph with outer face \( C \) of length \( p \leq 6 \). Assume that \( \Phi \) is a list assignment for \( G = (V, E) \) and \( S \subseteq V \) so that \( \Phi(v_i) = 1 \) for all \( v \in S \). Then if there is a \( \Phi \)-coloring of \( G \), we say that this coloring extends the coloring of \( S \).

Small Separating Cycle

Theorem 5. (Loeb) Suppose that \( G \) is a plane graph. Suppose that \( G \) is a plane graph. Suppose that \( G \) is a plane graph. Suppose that \( G \) is a plane graph. Suppose that \( G \) is a plane graph. Suppose that \( G \) is a plane graph. Suppose that \( G \) is a plane graph.

Distance

Theorem 2. Let \( G \) be a planar graph, \( W \subseteq V(G) \) so that \( d(u, v) \geq 3 \) for all \( u, v \in W \). Then a coloring of \( W \) can be extended to a 6-list-coloring of \( G \).

References


For Further Information

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