**Mathematics**

**Harvey Mudd College**

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### Introduction

Roughgarden et al. (2006) proposed social selection as a behavioral game theoretic model for sexual reproduction that incorporates both cooperation and competition. Players oscillate between playing competitively to maximize their individual fitness, leading to a Nash Competitive Equilibrium, and playing cooperatively to maximize a team fitness function, leading to a Nash Bargaining Solution.

#### Question

Individuals don’t play competitively all the time or cooperatively all the time. How do they switch between playing competitively and cooperatively?

#### Example

![Image 1: Peacock Wrasse: Mediterranean fish](image)

**Breeding season between April and June**

- Female strategies: spawning in nests (deposit) or spawning out of nests (broadcast)
- Male strategies: guarding nests (stay) or intercepting females on sea floor (search)
- Eggs laid in nests have higher chance of survival
- Spawning out of nests more common at beginning and end of season; spawning in nests more common at peak

#### Individual Play

- Individual play: players play strategies attempting to maximize own individual fitness.
- \( p_1(t), p_2(t) \) = proportion of time players 1 and 2 play their strategy A
- \( 1 - p_1(t), 1 - p_2(t) \) = proportion of time players 1 and 2 play their strategy B
- \( \omega_i(p_1, p_2) \) = expected individual fitness for player i given \( p_1, p_2 \)
- Rate of change at which player i changes playing strategy A:

\[
\frac{\partial p_i}{\partial t} = \frac{1}{\pi} \cdot \frac{\omega_i(p_1, p_2) - \omega_i(1 - p_i, 1 - p_i)}{\pi_i(p_1, p_2)} \cdot p_i(1 - p_i).
\]

Roughgarden et al. (2006) gives no relationship is between these two styles of play, so we created a model that would allow individuals to switch.

#### Team Play

- Both players attempt to maximize team fitness
- Team fitness defined as the product of each player’s individual fitness given the threat point
- \( x_{AA}, x_{AB}, x_{BA}, x_{BB} \) are proportions of the time the players jointly play AA, AB, BA, and BB
- Rate of change for a joint strategy \( x_{ij} \) (where \( i \neq j \))

\[
\frac{\partial x_{ij}}{\partial t} = \frac{1}{\pi} \cdot \left( \frac{\omega_{ij}(1, 1) - \omega_{ij}(p_i, p_j)}{\pi_{ij}(1, 1)} \right) x_{ij} (1 - x_{ij}).
\]

Model

Given a vector \( p \) that represents the proportion of time players 1 and 2 play strategy A, a 2x2 matrix \( x \) that represents the proportion of the time the team jointly plays AA, AB, BA, BB, a fitness matrix \( W \), a time step, and an initial proportion of time players play cooperatively, and some constant change amount chAm. For each time step up to 100,000, choose team play with probability \( a \), individual play with probability \( 1 - a \). Change \( a \) by a specific change amount chAm to see whether the style of playing improves either player’s fitness or not. If \( a \) becomes either 0 or 1 then we stop running the code.

### Expectations of Model

- We expected the model to predict for the peacock wrasse to:
  - Competitively play search-broadcast together at the beginning of the season
  - Play more cooperatively with stay-deposit mid-season
  - Revert to competition, playing search-broadcast again near the end of the season

#### Inputs

We used the values in Table 3.3, taken using data from Luttbeg and Warner (1999); Pallaoro and Jardas (2003); van den Berghe (1990).

#### Table 2: Peacock Wrasse Input Values

<table>
<thead>
<tr>
<th>Payoff Matrix</th>
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<th>teamProp</th>
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</table>

### Results

Results were mostly as expected. However, levels of cooperation were never very high. 2 shows the progression of the proportion of team play as time goes on. The x-axis represents time, and the y-axis represents the proportion of team play. Red lines represent the individuals ending up playing competitively the entire time, and blue lines represent the individuals playing cooperatively the entire time.

#### Conclusions

In our model, the wrasse develop from playing competitively search-broadcast to cooperatively playing stay-deposit and competitively playing search-broadcast, to competitively playing stay-deposit, and finally to competitively playing search-broadcast. This follows the general trajectory of breeding strategies for the male and female peacock wrasse, which suggests there may be mechanisms of cooperation and communication that act to cause peacock wrasse to switch strategies. More research needs to be done, especially as to what appropriate inputs to the model might be, as well as what generalized conclusions can be drawn about a model that switches between individual play and cooperative play.

### References


### Acknowledgments

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