Research Proposal:
Surfactant Driven Thin Film Fluid Flow

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1 Introduction

Surfactant-driven thin film flow has some important applications in industry. One of these applications involves modelling the respiratory mechanism in the human body. The alveolar wall is coated with a mucous membrane, on which exists a monolayer of organic surfactant. Unfortunately when babies are born without this surfactant the deficiency can be fatal. Present post-natal surfactant delivery mechanisms have a high success rate but there are some cases when drug delivery fails, perhaps due to the uneven distribution in the alveoli [1]. This research will focus on understanding these dynamics to help the pharmaceutical industry identify best practices for drug delivery.

2 Proposed Research

This thesis is intended to satisfy the requirements of both the physics and the math departments. My research will focus on modelling the surfactant driven flow using a system of coupled partial differential equations (Equation 2). In this system, $h$ represents the height of the fluid, $\Gamma$ represents the surfactant concentration, $\sigma$ is the surface tension while $\beta$ and $\delta$ are constants.

$$h_t + \nabla \cdot \left( \frac{1}{2} h^2 \sigma \right) = \beta \nabla \cdot \left( \frac{1}{3} h^3 \nabla h \right) - \kappa \nabla \cdot \left( \frac{1}{3} h^3 \nabla \nabla^2 h \right)$$  (1)

$$\Gamma_t + \nabla \cdot (h \Gamma \nabla \sigma) = \beta \nabla \cdot \left( \frac{1}{2} h^2 \Gamma \nabla h \right) - \kappa \nabla \cdot \left( \frac{1}{2} h^2 \Gamma \nabla \nabla^2 h \right) + \delta \nabla^2 \Gamma.$$  (2)

New experimental data collected in the last year does not agree with the numerical simulation of the model in Equation 2 [2]. One possible issue could be the choice of equation of state that relates surfactant concentration to surface tension. The equation of state captures the driving force in
Figure 1: The evolution of the equation of state. It was initially linear (Tsukunova), and Bull and Grotberg modified it in the 90’s. Recent experimental data collected at NC State is represented by the black curve. Note how the black curve resembles a hyperbolic tangent [5].

surfactant-driven dynamics in which Marangoni forces (surface stresses) arise as a result of surfactant concentration gradients.

In thin film models, the equation of state was initially assumed to be linear [2]. Later Borgas and Grotberg assumed it to be inverse-cubic [2]. More recently, experiments in a Langmuir trough at NC State University suggest that the relationship looks more like a hyperbolic tangent - flat sections in the beginning and end with a steep drop off in the middle (as seen in Figure 1). These experiments measure the surface tension for various surfactant concentrations, rather than exploring the effect of gradients. This raises a question about how the dynamics might differ in each of the three regions.

I will conduct experiments within these three regions (two flat, one steep) and between them to verify if the dynamics indeed follow this new equation of state. I will also investigate the new equation of state using numerical solutions. A simulation suite exists that solves the old model. I will learn how to use this suite and make changes to the model. Once we
have explored the new equation of state, I will run the experiment (and numerical simulations) with shear thinning substances such as xanthan gum instead of glycerol. The ultimate aim would be to model the spreading of surfactant on a visco-elastic fluid (which would more closely model the mucous of the lung).

3 Prior Research

This subject was first studied by Gaver, Grotberg and Jensen [3]. Gaver and Grotberg developed the accepted model (Equation 2) which is still in use today [4]. More recently, a new fluorescence imaging technique developed at NC State University has allowed us to compare the existing model with new data [5]. This has revealed significant discrepancies between the model’s predictions and the observed data. The model was simulated using software that was developed by Claridge et al. and is available at his Github repository [6].

I have taken Physics 154 (Fields and Waves), Math 180 (Introduction to PDEs), Math 164 (Scientific Computing) and CS 70 (Data Structures and Program Management). These courses should provide me with a good starting point for developing and analyzing a model for this system and numerically simulating it.

References


