

Preparing Students for the College Mathematics Experience

(A Personal Wish List)

By

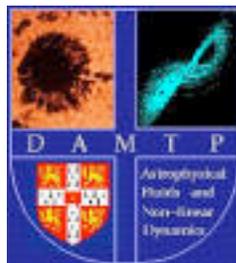
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HARVEY MUDD COLLEGE

Who am I ?

- [Andrew Bernoff's Homepage](#)



- I've spent time in departments of Math, Physics and various flavors of Engineering.

“Math lives and breathes through its applications.”

Who is Harvey Mudd ?

- Harvey Mudd College is located in Claremont, California.
- Specializes in Mathematics, the Sciences and Engineering.
- Roughly 700 students (all undergraduate) and 80 faculty.
- Median SAT score 1470 -- 25% HS valedictorians.
- Roughly half our students obtain graduate degrees.



Part I: Communicating Mathematics.

“No one would talk much in society if they knew how often they misunderstood others” -Goethe

Part II: Different styles of teaching and learning.

Part III: Tools and technology.

Neatness Counts!!

- Give students a template to work from.
- HMC Homework Template
 - Students tend to be better at copying an existing format than creating their own.

The image shows a sample of the HMC Homework Template. It features a header section for student information and a list of problems with designated spaces for problem statements and solutions.

Name
Class and Section
Homework Set Number
Due Date

Section 3.1 Problems 2, 5, 7

3.1.2 [problem statement]
[neatly written solution]

3.1.5 [problem statement]
[neatly written solution]

3.1.7 [problem statement]
[neatly written solution]

Honing Writing Skills

- Style points:
 - Let 5% of the credit for every assignment be for presentation, including neatness.
- Homework rewrites for credit:
 - Let students resubmit corrected assignments for up to half the points lost.
- **Extra Credit:** Mathematical text processing
 - LaTeX
 - Microsoft Equation Editor

$$\sum_{j=1}^N j = \frac{n(n+1)}{2}$$

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Real mathematicians use LaTeX !!!!

Writing Math

- Teach students to write. Mathematically. In whole sentences.
- Proposition: Show $M=n^2-1$ is not prime for n an integer greater than 2.
- Proof: $n^2-1 = (n-1)(n+1)$. $n>2$ so $(n-1),(n+1)>2$. M is the product of two numbers greater than 1 so M is composite.

Writing Math

- Teach students to write mathematically in whole sentences.
- Proposition: Show $M=n^2-1$ is not prime for n an integer greater than 2.
- Proof: Note that M can be written as the product of two factors,

$$M = n^2 - 1 = (n-1)(n+1).$$

As n is greater than 2, both factors of M , namely $(n-1)$ and $(n+1)$, are greater than 1, which implies M is composite. 😊

The Art of Proof

Like a Zen garden . . .



Succinct. Clear. Complete.

Proof Templates

For example, here is a template for induction:

To prove: **Proposition** -- show $A(n)$ is true for
 $n=1,2,3 \dots$

Proof: We wish to show $A(n)$.

Base Case ($n=1$): Show $A(1)$ is true here.

Induction Hypothesis: Show $A(n-1)$ implies
 $A(n)$ here.

Conclusion: By the Principle of Mathematical
Induction $A(n)$ is true for $n=1,2,3 \dots$ 😊

Proof Templates

To prove:

$$(A) \quad \sum_{j=1}^n j = \frac{n(n+1)}{2} \quad n = 1, 2, 3, \dots$$

Proof: We wish to show: $\sum_{j=1}^n j = \frac{n(n+1)}{2} \quad n = 1, 2, 3, \dots$

Base Case (n=1): For n=1, $\sum_{j=1}^1 j = \frac{1(1+1)}{2} = 1$.

IHOP: Suppose (A) is true for n. Then

$$\sum_{j=1}^{n+1} j = \left(\sum_{j=1}^n j \right) + (n+1) = \left(\frac{n(n+1)}{2} \right) + (n+1) = \frac{(n+1)[(n+1)+1]}{2}.$$

Which implies (A) is true for n+1.

Conclusion: By the PMI, (A) is true for n=1,2,3... .



Extra Credit: Projects and Presentations

- **Presentation is theater!!** Teach students to project and communicate with their audience.
- **Blackboard work:** Organize, use the whole board, and write horizontally, *not at an angle !!!*
- **Flair & Humor:** Develop a style and have a little fun. 😊
- **Power point and other technology:** Incorporating technology is great as long as it doesn't become the message.

Summary:

Communicating Mathematics

- Neatness Counts !!
- Reward clear, concise mathematical writing.
- Writing templates can set an example.
- Encourage students to get comfortable speaking in class; presenting homework solutions is a great way to start.

Part I: Communicating Mathematics.

Part II: Different styles of teaching and learning.

– *“Education is what survives when what has been learned has been forgotten.”* B.F. Skinner

Part III: Tools and technology.

Teaching Students to Work Independently

“Education is what survives when what has been learned has been forgotten.” B.F. Skinner

- Developing the ability to teach oneself new skills is perhaps the most important form of education.
- The ability to read a book, navigate a library, and find information on the WWW will help a student in many subjects, and will last long beyond a given semester.

Independent Work - Some Ideas

- **Throw them in the Deep End !!**
 - Give them a section in the book to read on their own, and then ask them to explain it to you (or the class).
- **Treasure Hunt.**
 - Ask them an open ended question; perhaps let them try to solve it in class, then send them out to find the answer
 - “How many gas stations are there in the U.S. ?”*
 - “What is the largest prime number known?”*
- **Teach me something new.**
 - Ask them to find you something interesting about a topic:
 - “Tomorrow I want you to tell me a fun fact about ellipses.”*

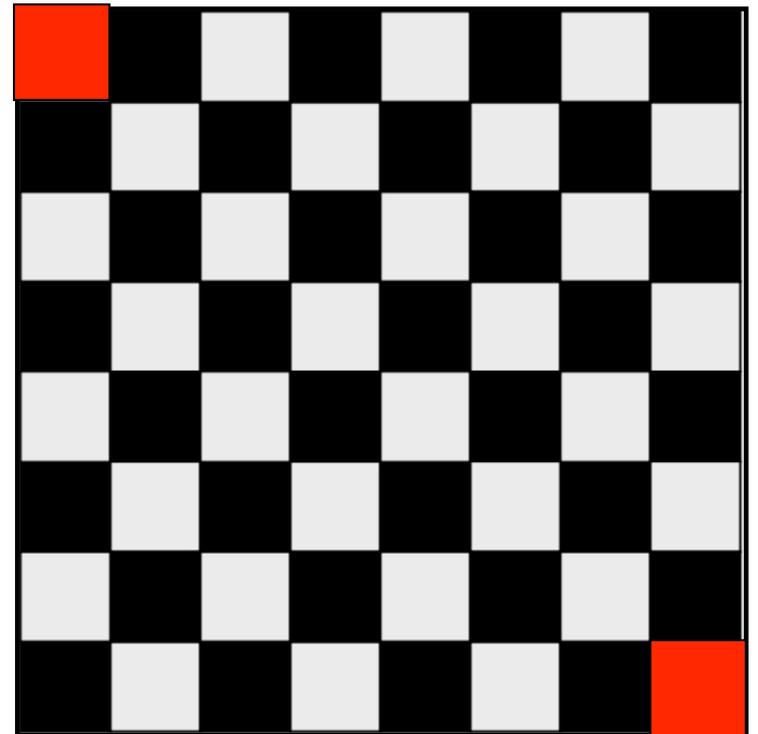
Small Group Work

- **Why?** Throughout college and their professional career, students will be working in small groups to study and work. The ability to work well collaboratively is an essential skill.
- **What is a good small group problem?** Generally, it is a problem that needs interpretation and has a certain subjectively flavor.
 - Bad:** *“What is the largest prime number known?”*
 - Good:** *“How many gas stations are there in the U.S. ?”*
 - Better:** *“See how many different ways you can estimate how many gas stations are there in the U.S. Compare your answers and decide which one you think is the best estimate. See if you can find the answer on the web -- how does it compare with your estimates?”*

Dominos and Tiling: A small group activity

Q: Can you cover an 8-by-8 checkerboard whose corner squares have been removed with 2-by-1 dominos?

A: No, because you have 32 black squares and 30 white squares, and each domino must cover a black and white square.



How can you generalize this problem?

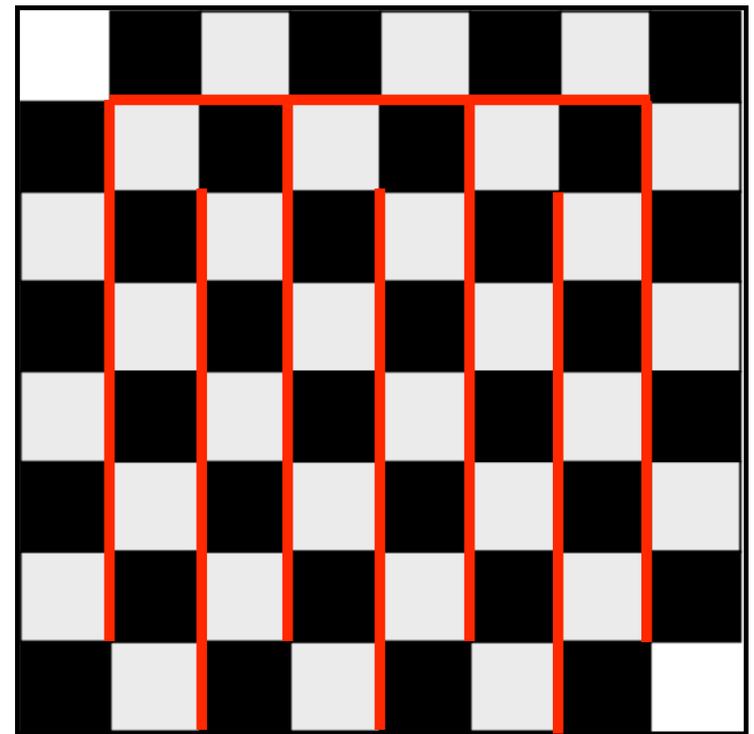


Dominos and Tiling: Conjecture and Proof

Generalizations:

1. You can't cover an 8-by-8 board if you remove two squares of the same color.
2. You can cover an 8-by-8 with two squares removed if they are of different colors.
3. =(1.+2.) You can cover an 8-by-8 board with two squares removed if and only if they are of different colors.

“Necessary and Sufficient”



Volume of a Pear: A small group activity

Q: What is the volume of the pear?

Possible Answer methods:

- Compare to a sphere.
- Compare to a cone + sphere
- Slice it into cylinders.
- Volume of rotation.
- Simpson's rule.
- Play "Archimedes"

How can you generalize this problem?



The Dangers of Small Groups

- **Dominant personalities:** Extroverts and introverts have their place in a group, but sometimes they need help getting along. Some ideas:
 - Appoint a team leader.
 - Have a speaking wand.
- **Gender issues:** There's something to that whole Mars-Venus thing. Things to try:
 - Pair women up.
 - Single gender groups.

Remember group work is a learning experience; building collaborations and dealing with conflict is part of that experience!!!!

(I need some) Help!

- Knowing **when to ask** for help.
 - *“I’ve been spinning my wheels on this problem for an hour.”*
 - *“Well, I did all the homework, but none of them agree with the answers in the book.”*
- Knowing **how to find** help.
 - Study groups: For many students building a good study group is an essential for surviving college.
 - Office Hours and Tutorials.
- Knowing **how to ask** for help:
The care and feeding of your professors.
 - Politeness and respect.
 - Work with your professors, not against them.
“I want you to pass the course and you want to pass the course. Why can’t we get along?”

Summary:

Styles of Learning

- Encourage students to work independently; get them to read the textbook and explore subjects on the WWW.
- Working in small groups is a college survival skill. Group work in class normalizes this.
- Good small group problems tend to be open-ended and have multiple avenues of attack.
- Students should learn when, where and how to find help. Create resources and encourage them to take advantage of them.

Part I: Communicating Mathematics.

Part II: Different styles of teaching and learning.

Part III: Tools and technology.

–1980: *“An hour in the library can save you a month of research.”*

–2000: *“A minute on the web can save you an hour in the library.”*

Web resources

“A minute on the web can save you an hour in the library.”

- **Search Engines:** These are great for getting started.

Google

“What is the largest prime number known?”

“Tell me a fun fact about ellipses.”

“What is known about tiling a checkerboard with dominos?”

Web resources

- [Mathworld](#) is Eric Weisstein's wonderful online encyclopedia for more advanced students.
- [Math Fun Facts](#) is Francis Su's website with many amusing math facts for students of all abilities.
- [Math Forum](#) is a good starting place for K-12 resources.
- The [Mathematics Association of America](#) has many articles and resources, some suitable for K-12.

WWW: The pitfalls of technology

- Things your students should know: **Plagiarism**
“While it was long believed that Riemann's hypothesis was the result of deep intuition on the part of Riemann . . .”

[Google](#)

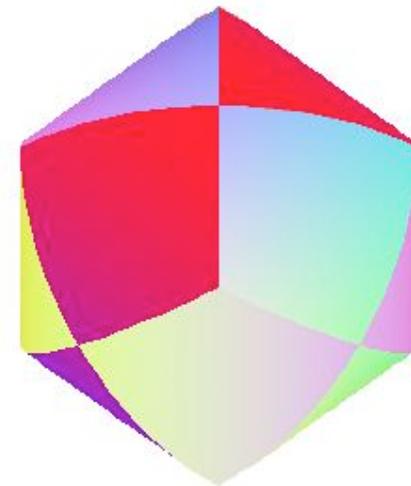
- Things you should know:
 - Cramster
 - Text messaging and the wireless revolution.

MAPLE

MAPLE is a graphical and algebraic interface.

It can:

- Factor
- Integrate and differentiate
- Graph curves and surfaces.



Tools like MAPLE can improve students visualization skills, and let them check their algebra and calculus.

Summary: Tools & Technology

- Help students learn to use the web as a research resource.
- Construct a reference webpage of good sites for them to visit.
- Think about incorporating MAPLE or other graphical/algebraic programs into your curriculum, both for demonstrations and for students to use as a tool.

A final plea . . .

The world is a scary place at the moment . . . math can help, by making sure students have rational fears.

“Am I more likely to be on a plane that is hijacked, or to be killed by a drunk driver?”

“If I start smoking now, what are the chances I will die of lung cancer or heart disease?”

“What are the chances somebody I know has HIV?”

“Math lives and breathes through its applications.”