

Problem Solving in a Small Group Setting

Here are three problems for you to work on in small groups (two to six people).

You might start by trying to guess the answer to the problems by trial and error. There are also hints (in small print) at the bottom of the page.

After you think you have a solution, you might ask yourself the following questions:

- Have I considered all cases? Can I prove that the answer I have is right?
- How can I best organize my argument for an oral presentation? How can I organize my solution so that it is succinct, clear, and compelling?

At the end of the session we will present the solutions to each other and discuss the dynamics of working in groups.

Problem 1: (a) Given a 5×5 chessboard with one square removed, we wish to tile the rest of the board with 1×3 blocks. For which removed squares is this possible, and for which is it not?

(b) Suppose we now wish to tile an 8×8 chessboard with one square removed, with 1×3 blocks. For which removed squares is this possible, and for which is it not? How about an $N \times N$ chessboard?

Problem 2: On a tropical island live R red chameleons, B blue chameleons and Y yellow chameleons. When two chameleons of different color meet they change to the third color. For what triples (R, B, Y) can all the chameleons eventually be the same color?

Problem 3: (a) A deck of 52 playing cards is printed on a large rectangular sheet consisting of four rows of thirteen cards each. What is the minimum number of linear cuts needed to separate the cards, assuming that piling (but not folding) is permitted? Is there a more efficient arrangement of the cards on a rectangular sheet that reduces the number of cuts?

(b) Can you generalize this problem to $M \times N$ sheet? Check your answer for a 9×7 sheet with 63 cards.

Hints:

1. Can you color the board with three colors in a way that is helpful?
2. Start with three chameleons and work your way up.
3. For every cut, each card is on the right or the left side. What does this tell you?