Introduction

In the United States, 641,086 cases of AIDS were reported to the Centers for Disease Control and Prevention in 1997.

In 2000, there were 774,467 cases reported and 886,575 in 2002.

As of December 31, 2000, 21% of infection occurred among females.
Of new infections among women in the United States, CDC estimates that approximately 75 percent of women were infected through heterosexual sex.

Thompson’s Model excludes heterosexual and bisexual populations.
Questions asked

- Can Thompson’s model be improved to reflect the new trend?
- How the values of parameters in the model affect the progression of AIDS?
- What can be done to stop the AIDS epidemic?
Thompson’s Assumption

- Uniform Mixing
- Interactions are from man to man through sexual contact
Variables and Parameters

\( W \) = number of intermediates
\( X \) = number of susceptibles
\( Y \) = number of infectives
\( K \) = average number of sexual contacts per month per person
\( \alpha \) = probability of contraction AIDS through a single contact
\( \beta \) = ratio of \( k \) value of active to less active population
\( \gamma \) = transformation rate from \( W \) to \( Y \)
\( \mu \) = emigration rate
\( \lambda \) = immigration rate
\( \psi \) = marginal AIDS death rate
\( \rho \) = a high contact fraction
Thompson’s Model

\[
\frac{dW_1}{dt} = \frac{k \ X_1 (Y_1 + Y_2)}{X_1 + Y_1 + (Y_2 + X_2)} - (\mu + \rho) W_1
\]

\[
\frac{dW_2}{dt} = \frac{k \ X_2 (Y_1 + Y_2)}{X_1 + Y_1 + (Y_2 + X_2)} - (\mu + \rho) W_2
\]

\[
\frac{dY_1}{dt} = W_1 - (\mu + \rho) Y_1
\]

\[
\frac{dY_2}{dt} = W_2 - (\mu + \rho) Y_2
\]

\[
\frac{dX_1}{dt} = -\frac{k \ X_1 (Y_1 + Y_2)}{X_1 + Y_1 + (Y_2 + X_2)} + (1 - \rho) - \mu X_1
\]

\[
\frac{dX_2}{dt} = -\frac{k \ X_2 (Y_1 + Y_2)}{X_1 + Y_1 + (Y_2 + X_2)} + \rho - \mu X_2
\]
New Assumptions

- Uniform Mixing
- Only male infective population is infectious: infections are man to man or man to woman through sexual contact
- Male to female ratio in the United States population is 1:1
- All parameter values are the same for male and female population except $\kappa \alpha$. 
Revised Model

\[
\begin{align*}
\frac{dW_{f1}}{dt} &= \frac{kX_f(Y_{m1} + Y_{m2})}{X_{f1} + Y_{f1} + (Y_{f2} + X_{f2})} - (\mu + \mu)W_{f1} \\
\frac{dW_{f2}}{dt} &= \frac{kX_f(Y_{m1} + Y_{m2})}{X_{f1} + Y_{f1} + (Y_{f2} + X_{f2})} - (\mu + \mu)W_{f2} \\
\frac{dY_{f1}}{dt} &= W_{f1} - (\mu + \mu)Y_{f1} \\
\frac{dY_{f2}}{dt} &= W_{f2} - (\mu + \mu)Y_{f2} \\
\frac{dX_{f1}}{dt} &= -\frac{kX_f(Y_{m1} + Y_{m2})}{X_{f1} + Y_{f1} + (Y_{f2} + X_{f2})} + (1 - p) - \mu X_{f1} \\
\frac{dX_{f2}}{dt} &= -\frac{kX_f(Y_{m1} + Y_{m2})}{X_{f1} + Y_{f1} + (Y_{f2} + X_{f2})} + p - \mu X_{f2}
\end{align*}
\]
Choosing Parameters

\( k_\alpha = 0.023 \)
\( \beta = 1/24 \)
\( \mu = 0.00417 \)
\( \lambda = \mu X \) (to keep the total population constant)
\( \tau = 5 \)
\( \gamma = 1/36 \)
\( \rho = 0.2 \)
\( k_\alpha_{\text{female}} = 0.001 \)
Simulations

Number of Susceptibles

Population

Number of intermediates and infectives

Population

time (months)

Simulations
# Improvement

<table>
<thead>
<tr>
<th></th>
<th>Surveyed</th>
<th>New Model</th>
<th>Thompson’s Model</th>
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</thead>
<tbody>
<tr>
<td>1997</td>
<td>641,086</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>744,467</td>
<td>763,300</td>
<td>823,130</td>
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<td>2002</td>
<td>886,575</td>
<td>1,272,000</td>
<td>1,399,100</td>
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Results
Results
Results
Conclusions

- New model can predict AIDS progression better.
- Smaller values of $\kappa \alpha$ and $\lambda$ can slow down or stop AIDS progression.

Public education:
  - For general public – to reduce number of sexual contacts and have safe sex using protection.
  - For infectives – to restrain from having sexually active life-style.
Future Work

- Can the model be improved by investigating different age groups?
- How AIDS treatment affect the model and should AIDS be treated?
- How is the AIDS epidemic worldwide interacting with the U.S. epidemic?
Questions?