Modeling of AIDS Epidemics

Introduction

In the United States, 641,086 AIDS cases were reported to Centers for Disease Control and Prevention in 1997. The number of AIDS cases have been increasing ever since, reaching 886,575 cases in 2002. It was usually assumed that AIDS occurred mostly among male homosexual population. However, in 2000, 21% of infective population were females. One of AIDS epidemic models is Thompson’s which is discussed in his article, “AIDS: The Mismanagement of an Epidemic” published in 1989. Thompson assumes that the majority of AIDS infective population are male homosexuals. However, the percentage of infective females becomes significant, Thompson’s model needs to be revised.

Questions to Ask

• Qualitative: Can Thompson’s model be improved to reflect the new trend of increasing numbers of infected females? Also, what can be done to stop the AIDS epidemic?

• Quantitative: How the values of parameters in the model affect the progression of AIDS and what values of parameters stop the AIDS epidemic?

Assumptions

• Uniform mixing – no spatial discrepancies.
• AIDS infections are man to man or man to woman through sexual contacts.
• Male to female population ratio in the U.S. is 1:1.
• All parameter values are the same for male and female population except $\kappa a$.

Model

$X = \text{susceptibles}$

$W = \text{intermediates}$

$Y = \text{infectives}$

Subscripts of 1 & 2 = sexually less active and high active, respectively

Subscripts of m & f = male and female population, respectively

This is a diagram of interactions among subpopulations. The single-headed arrows indicate the state transitions and the double-headed arrows indicate sexual interactions and transmission pathway of AIDS.

$\frac{dW}{dt} = \frac{k a X_{1} (Y_{1} + Y_{2})}{X_{1} + Y_{1} + \tau (Y_{1} + X_{1})} - (\beta + \mu)W_{1}$

$\frac{dW}{dt} = \frac{k a X_{2} (Y_{1} + Y_{2})}{X_{2} + Y_{1} + \tau (Y_{1} + X_{2})} - (\beta + \mu)W_{2}$

$\frac{dY_{1}}{dt} = \beta W_{2} - (\mu + \gamma)Y_{1}$

$\frac{dY_{2}}{dt} = \beta W_{1} - (\mu + \gamma)Y_{2}$

$\frac{dX_{1}}{dt} = \frac{-kaX_{1}(Y_{1}+Y_{2})}{X_{1}+Y_{1}+\tau(Y_{1}+X_{1})}+(1-\rho)\lambda - \mu X_{1}$

$\frac{dX_{2}}{dt} = \frac{-kaX_{2}(Y_{1}+Y_{2})}{X_{2}+Y_{1}+\tau(Y_{1}+X_{2})}+(1-\rho)\lambda - \mu X_{2}$

Parameters:

• $\kappa$ – average number of sexual contacts per month per person
• $\mu$ – probability of contact causing AIDS
• $\tau$ – ratio of $\kappa$ value of active to less active population
• $\beta$ – transmission rate from W to Y
• $\rho$ – emigration rate
• $\lambda$ – immigration rate
• $\gamma$ – marginal AIDS death rate
• $\rho$ – a high contact fraction

The equations on the left side is Thompson’s model which is adopted as the model for male population. The equations on the right side is a revised version of Thompson’s model and are used for female population. The model for female population reflects the transmission of AIDS from infective male to female susceptible population.

Results

<table>
<thead>
<tr>
<th>Year</th>
<th>New Model</th>
<th>Thompson’s</th>
<th>Surveyed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>775,470</td>
<td>641,086</td>
<td>744,467</td>
</tr>
<tr>
<td>2000</td>
<td>2,343,500</td>
<td>7,420,600</td>
<td>868,575</td>
</tr>
<tr>
<td>2002</td>
<td>1,277,600</td>
<td>7,420,600</td>
<td>868,575</td>
</tr>
</tbody>
</table>

Using statistics from 1997 as initial conditions, AIDS populations are estimated with the new model and Thompson’s model.

Conclusion

By comparing to the surveyed AIDS population, the new model proved to be more accurate than Thompson’s model. Small values of $\kappa a$ and $\tau$ are shown to slow down the rate of AIDS transmission or even to stop the AIDS epidemic. To achieve those values, a possible approach is public education for both of general and infective population. $\kappa a$ value can be lowered greatly when general or susceptible population have less number of sexual contacts and safe sex with protection. Moreover, by restraining infective population from having highly sexually active lifestyles, $\tau$ value can be lowered. When these goals are achieved, it is possible to stop the AIDS epidemic.