Assignment 2: Plotting a Circle and Whales and Krill

Math 164 - Scientific Computing
Spring Semester 2003
Due Date: Tuesday, February 11, in class.
Prof. de Pillis

February 13, 2003

1. Plotting a Circle. With your team member, write up solutions to the following problems:

(a) Implement (in Matlab) the Forward Euler integration method discussed in class to plot a circle, based on solving the system

\[ \begin{align*}
    x'(t) &= -y(t), \quad y(0) = 0 \\
    y'(t) &= x(t), \quad x(0) = 1
\end{align*} \]

with \(0 \leq t \leq 2\pi\).

(b) Develop and implement (in Matlab) an integration scheme that is different from the Forward and Backward Euler difference schemes discussed in class.

(c) Attempt to predict analytically how well your new scheme will behave. Note that this system of ODEs is linear.

(d) Try different discretization step sizes for both the Forward Euler scheme and for your new and improved scheme, and tabulate the errors in each case. Can you tell from the pattern of the errors what the order of the Euler scheme versus your scheme is? (Note: The “order” of a numerical scheme is the integer \(p\) for which the local discretization error is \(O(h^p)\). You can get a sense for this numerically in the following way: Run your scheme at a certain discretization step size \(h\), figure out the error, halve the step size and run it with \(h/2\), halve it again and run it with \(h/4\), etc. If your error gets halved every time \(h\) is halved, your error is \(O(h)\), and is first order. If your error shrinks faster than \(h\), it may be \(O(h^2)\) (second order) or better.)

2. Whales and Krill: Classroom Model Recall from class the predator-prey model developed to simulate the interaction of whales with krill:

\[ \begin{align*}
    \frac{dK}{dt} &= (a - bW)K \\
    \frac{dW}{dt} &= (-m + nK)W
\end{align*} \]

where \(K\) is the krill population, \(W\) is the whale population and \(a, b, m, n > 0\). In the experiments below, use the following values: \(a = 1.0, b = 0.5, m = 0.75, n = 0.35, K_0 = 5.0, W_0 = 3.0\). Run your simulation from time 0 to time 50, that is \(t_{\text{init}} = 0, t_{\text{final}} = 50\). With your team member, write up solutions to the following problems:
(a) In Matlab, write your own Forward Euler scheme to solve this system of ODEs. Take 2000 time steps between $t_{init} = 0$ and $t_{final} = 50$. Plot the results, one plot with Krill on the x-axis and Whales on the y-axis, and one plot with time on the x-axis and both Whales and Krill on the y-axis.

(b) Now implement your improved integration scheme from above to solve the two-population system of ODEs. Plot the results in the same way.

(c) Answer the question: Now that the ODE system you are solving is nonlinear, what becomes different about the analysis you would apply to try to predict how well your scheme will behave? Is it still possible to do analysis similar to the one in the linear case?

(d) Once again, run your scheme with decreasing step-sizes. Does the order of your method appear to be the same as in the linear case? Recall from class: The implicit form of the solution to this system of ODEs is given by

\[ a \ln W - bW = nK - m \ln K + c, \]

where $c$ is a constant depending on initial conditions.

3. **Whales and Krill: Extended Model** Add the following new features to the system of Whale-Krill ODEs to enhance your model:

(a) Modify the model to reflect the effects of harvesting the krill at some rate (that is, add a new “harvesting” term). Run numerical simulations. What can happen to the Whale-Krill populations? Plot your results as above.

(b) Modify the model to account for internal competition for resources (overcrowding): both whale-whale competition and krill-krill competition. Run your numerical simulations both with and without harvesting. What happens? Plot your results as above.

4. **Guidelines for this homework.** All write-ups should be text-formatted (no handwritten homework papers, please). Make sure to include illustrative graphics. Hand in all three parts together.

In addition your write-up of the report to be handed in, develop a five-minute presentation of your results to give in class (using, e.g., your overhead transparencies).

**MOST IMPORTANTLY: If you are confused about any aspect of this assignment, or would like help, some hints, or guidance, then ask me (come to my office, call on the phone, use e-mail...although my e-mail response time is slower). I will be happy to help you out.**