For centuries, people have noted that the face of the sun is not constant or uniform in appearance, but that darker regions appear at random locations on a cyclical basis. In 1984 Rudolf Wolfer proposed a rule that combined the number and size of these dark spots into a single number. Using archival records, astronomers have applied Wolfer’s rule and determined sunspot numbers back to the year 1700. Today these are measured by many observatories and the worldwide distribution of the data is coordinated by the Swiss Federal Observatory on a daily, monthly and yearly basis. Sunspot activity is cyclical and variation in the Wolfer numbers has been correlated with weather and other terrestrial phenomena of economic significance. This accounts for the continuing interest in them. This problem is adapted from “Numerical Methods and Software” by Kahaner, Moler, Nash, Prentice Hall, 1989.

1. Computation

- The file
  
  http://www.math.hmc.edu/~depillis/MATH164/sunspot.dat
  
  contains the average Wolfer sunspot number for each year from 1700 to 1986. Import these data into Matlab. Plot the data.

- Use fft on these data. In Matlab, the function fft performs the complex fast Fourier transform, assuming complex data. You will need to use the functions real and imag to extract the coefficients $a_k$ and $b_k$ of the real and imaginary parts, respectively.
  
  - Plot the power as a function of frequency, i.e., the periodogram, omitting the $k = 0$ term.
  - Is there a single frequency that dominates?
  - What is the period of the dominant cycle?
  - Was this apparent in the original data?
  - The sunspot data are only given through 1986. Given the trends you see in the data, predict in which years (up through 2003) sunspot activity should have been at a maximum.
  - Extra Credit: Find and document data that either confirm or contradict your prediction of when maximum sunspot activity should have taken place between 1986 and 2003.

- Set all frequency coefficients $a_k$ and $b_k$ to zero, except for $a_0$, and the two other most dominant frequencies (for those frequencies, keep both $a_k$ and $b_k$). Now use ifft to perform the inverse transform and get back the Wolfer numbers with only the dominant frequencies and the average value present. Plot these data along with the original Wolfer numbers and compare. How well
do just two frequencies capture the original data? Do the same using $a_0$ and the four other most dominant frequencies. How well do the four frequencies capture the original data? Do the same with $a_0$ and the eight other most dominant frequencies. How well do the eight frequencies capture the original data?

- Instead of using the full original data set, use every other value and fft to compute the discrete Fourier transform. Plot the power as a function of frequency, omitting the $k = 0$ term. Keep decreasing the sampling rate: try this again using every third value, then every fourth value, etc., until aliasing causes the transform to be invalid. When does this occur?

2. **Individual Research** Find another set of data that can be analyzed using a discrete fourier transform (it may be visual data, sound data, data tracking certain phenomena over time, etc.). Perform the appropriate transforms on the data, and discuss the analysis of your results. Include enlightening graphics.

3. **Discussion**

   Turn in all plots mentioned, along with a write-up including a description of what you did and an explanation of your results (including answers to the specific questions asked).

4. **Guidelines for this homework.**

   **Individual Work:** This homework project is to be done individually. You may discuss possible solutions and approaches with other students and the professor, but the final work and write-up is to be your own.

   **Format of Write-up:** All write-ups should be text-formatted (no handwritten homework papers, please). Make sure to include illustrative graphics. **Format of writeup:** Please follow the guidelines given in the *Homework Template* at the beginning of the semester. The template can also be found on the class web page.

   **Class Presentation:** In addition your write-up of the report to be handed in, develop a five-minute presentation of your results to give in class. You do not have to present every part of your write-up. Instead, **your presentation should focus mainly on the new data you found, and your analysis**, the new question you asked, and the conclusions that you came to, along with justification. You may give an on-line presentation (PowerPoint, or Post-Script slides, or PDF-slides), or use text-formatted overhead transparencies. Hand-written presentations are not viewed favorably. However, if your presentation must be hand-written, please make sure your writing is extremely neat and legible.

   **Getting Help:** If you are confused about any aspect of this assignment, or would like help, some hints, or guidance, then ask me (come to my office, call on the phone, use e-mail...although my e-mail response time is slower). I will be happy to help you out.