

Math 11, Fall 2007

Midterm Exam

September 28, 2007

This test is a **closed-book, closed-notes, 50-minute in-class** test. **No calculators or computers** are allowed. You must also **not discuss** any aspect of this test until noon on Friday, September 28.

Neatness counts. Please justify your steps, show necessary work, and include sentences when appropriate. Partial credit will be considered. Some points will be awarded for good writing style.

Name: _____

Section: Benjamin 10am 11am Jacobsen 10am 11am

1	20
2	20
3	15
4	20
5	20
Style	5
Total	100

Problem 1. (20 points) Use mathematical induction to prove

$$1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + N \cdot 2^N = 2 + (N - 1)2^{N+1} \quad \text{for } N \geq 1.$$

Be sure to write your answer in the correct form, explaining the logical steps – part of the grade is based on this.

Problem 2. (20 points) Use an ε - δ argument to prove $\lim_{x \rightarrow 2} (x^3 - 2x + 8) = 12$. Start your proof as follows (with the blanks filled in by you):

Proof: Let $\varepsilon > 0$ be given. We must find _____ such that for _____ it follows that _____ $<$ _____. Choose $\delta =$ _____. Then it follows that

Space for scratch work:

Problem 3. (15 points) Find $\lim_{x \rightarrow 0} \frac{\sin^2 x - x^2}{x^4}$. Justify your steps in computing the limits.

Problem 4. (20 points) Find $f'(x)$ for the following functions (wherever $f(x)$ is defined):

(a) $f(x) = \ln(\cos x)$

(b) $f(x) = 2^{\sqrt{x}}$

Problem 5. (20 points)

Part A. Suppose $f(x)$ satisfies $f'(x) = f(x)$ for all $x \in \mathbb{R}$. Show $f(x) = Ce^x$ for some constant $C \in \mathbb{R}$. Hint: Consider the function $g(x) = \frac{f(x)}{e^x}$.

Part B. Suppose $f(x) = \int_0^x f(t) dt$. Prove $f(x) = 0$ for all $x \in \mathbb{R}$.

Note: Even if you did not finish Part A, you can still use it for Part B.