

Dummit & Foote (4.5) 1, 4, 5, 17.

Let G be a finite group and let p be prime.

4.5.1 Prove that if $P \in \text{Syl}_p(G)$ and H is a subgroup of G containing P then $P \in \text{Syl}_p(H)$. Give an example to show that, in general, a Sylow p -subgroup of a subgroup of G need not be a Sylow p -subgroup of G .

4.5.4 Exhibit all Sylow 2-subgroups and Sylow 3-subgroups of D_{12} and $S_3 \times S_3$.

4.5.5 Show that a Sylow p -subgroup of D_{2n} is cyclic and normal for every odd prime p .

4.5.17 Prove that if $|G| = 105$ then G has a normal Sylow 5-subgroup and a normal Sylow 7-subgroup.