Math 171 - Abstract Algebra I, Section __
HW #12
10/21/10

Dummit & Foote (7.3) 17, 24. (7.4) 7, 8.

7.3.17 Let R and S be nonzero rings with identity and denote their respective identities by 1_R and 1_S. Let $\phi : R \to S$ be a nonzero homomorphism.

(a) Prove that if $\phi(1_R) \neq 1_S$ then $\phi(1_R)$ is a zero divisor of S. Deduce that if S is an integral domain then $\phi(1_R) = 1_S$.

(b) Prove that if $\phi(1_R) = 1_S$ then $\phi(u)$ is a unit in S and $\phi(u)^{-1} = \phi(u^{-1})$.

7.3.24 Let $\varphi : R \to S$ be a ring homomorphism.

(a) Prove that if J is an ideal of S then $\varphi^{-1}(J)$ is an ideal of R. Apply this to the special case when R is a subring of S and φ is the inclusion homomorphism to deduce that if J is an ideal of S then $J \cap R$ is an ideal of R.

(b) Prove that if φ is surjective and I is an ideal of R then $\varphi(I)$ is an ideal of S. Give an example where this fails if φ is not surjective.

7.4.8 Let R be an integral domain. Prove that $(a) = (b)$ for some elements $a, b \in R$, if and only if $a = ub$ for some unit u of R.