Math 171 - Abstract Algebra I Section _
HW #22
12/9/10

1. (D & F 10.1, 1) Let R be a ring with identity and let M be an R-module. Prove that $0 \cdot m = 0$ and $(-1) \cdot m = -m$ for all $m \in M$

2. (D & F 10.1, 9) Let N be a submodule of an R-module M. Then the annihilator of N in R is defined to be

$$\mathrm{Ann}_R(N) = \{r \in R \mid rn = 0 \text{ for all } n \in N\}.$$

Show that $\mathrm{Ann}_R(N)$ is an ideal of R.

3. Let $R = \mathbb{C}[S_2]$. Then the complex vector space V spanned by v_1 and v_2 is an R-module under the following action:

$$(ae + b(12)) \cdot (cv_1 + dv_2) = (ac + bd)v_1 + (ad + bc)v_2,$$

where e denotes the identity element of S_2. (This module corresponds to the group action where (12) switches v_1 and v_2.)
Find all submodules of V. (Hint: Argue that since $\mathbb{C}[S_2]$ contains elements of the form ze for all z, a submodule of V is also a subspace of V.)