Math 171 - Abstract Algebra I
HW #4
9/16/10

Dummit & Foote 1.6 # 7, 13, 14, 23

1.6.7 Prove that D_8 and Q_8 are not isomorphic.

1.6.13 Let G and H be groups and let $\varphi : G \to H$ be a homomorphism. Prove that the image of φ, $\varphi(G)$, is a subgroup of H (cf. Exercise 26 of Section 1). Prove that if φ is injective then $G \cong \varphi(G)$.

1.6.14 Let G and H be groups and let $\varphi : G \to H$ be a homomorphism. Define the *kernel* of φ to be $\{g \in G \mid \varphi(g) = 1_H\}$ (so the kernel is the set of elements in G which map to the identity of H, i.e., is the fiber over the identity of H). Prove that the kernel of φ is a subgroup (cf. Exercise 26 of Section 1) of G. Prove that φ is injective if and only if the kernel of φ is the identity subgroup of G.

1.6.23 Let G be a finite group which possesses an automorphism σ (cf. Exercise 20) such that $\sigma(g) = g$ if and only if $g = 1$. if σ^2 is the identity map from G to G, prove that G is abelian (such an automorphism σ is called *fixed point free* of order 2). [Show that every element of G can be written in the form $x^{-1}\sigma(x)$ and apply σ to such an expression.]