Exam I
Math 171

This exam is to last for no more than **three hours**. It is closed-book and closed-notes. You should feel free to quote, without proof, results from class and the homework. You should neither receive nor give any assistance, and you should **not discuss** any aspect of this exam until **all** of the exams have been returned to your instructor.

Please keep in mind that each of these questions should be used as a vehicle to **demonstrate your understanding** of the material presented in this course. Lastly, please be sure to **justify your remarks**, and to strive for **clarity, cohesiveness, and transparency** throughout the entire exam.

1	10
2	10
3	10
4	10
5	10
6	10
Style	
Total	60

Name: ____

As for everything else, so for a mathematical theory: beauty can be perceived but not explained.

— Arthur Cayley

Problem 1. Consider the following subset of 2×2 matrices with real entries:

$$G = \{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbb{R}, a \neq 0 \}.$$

Is G a group under matrix multiplication?

Problem 2. If H and K are normal subgroups of G, and $H \cap K = \{e\}$, prove that G is isomorphic to a subgroup of $G/H \times G/K$.

Problem 3. Let G be a group. If $H = \{g^2 \mid g \in G\}$ is a subgroup of G, prove that H is a normal subgroup of G.

Problem 4. Prove that no group can have exactly two elements of order 2.

Problem 5. Suppose that G is a group that has exactly one nontrivial subgroup. Prove that G is cyclic and $|G| = p^2$ for some prime p.

Problem 6. Let G be a group of order 60. If the Sylow 3-subgroup is normal, show that the Sylow 5-subgroup is normal.