1. Let \( f(x) = -\frac{13}{12}x^2 + \frac{49}{12}x + 1. \) Verify that \( x = 4 \) is a periodic point. What is its prime period? What are the other points in its orbit? Use the multiplier theorem to determine if this a stable or unstable periodic orbit.

2. Define the following dynamical system: let the state space \( S = [0, 1] \), all numbers between 0 and 1. Let \( f \) be the function which takes the decimal representation, cuts off the first digit, and shifts everything left. [Example: \( f(0.1415926...) = (.415926...) \)].
   (a) What are all the fixed points of \( f \)? (In other words, which numbers which remain unchanged after applying \( f \) once?)
   (b) Describe all the periodic points of \( f \) of prime period 2?
   (c) Describe all points of \( f \) which are eventually periodic of any period? (Eventually periodic means that the iterates eventually repeat, though they may or may not repeat at first.)

3. Graph the following functions for a range of values of the parameter \( c \) to determine whether the bifurcation that occurs is a transcritical, tangent, or period-doubling bifurcation. Using Discrete Tool, estimate the precise value of \( c \) in each case where the bifurcation occurs.
   (a) \( f(x) = cx^2 \), as \( c \) varies between \( c = .3 \) and \( c = .4 \).
   (b) \( f(x) = x^2 - c \), as \( c \) varies between \( c = .6 \) and \( c = .9 \). [Focus on the fixed point that is smaller and negative.]
   (c) \( f(x) = cx(1-x)^2 \), as \( c \) varies between \( c = .8 \) and \( c = 1.2 \). [Focus on what happens near the fixed point at 0.]

4. Use Discrete Tool to:
   (a) estimate the location of the first 4 period-doubling bifurcations for \( f(x) = x^2 - \lambda \). For instance, \( \lambda_1 = .75 \). What is \( \lambda_2, \lambda_3, \lambda_4 \)?
   (b) estimate the location of the period 3 “window” in the chaotic region.

5. Water is dripping from a leaky faucet. The time between drips can be modeled by some (unknown) discrete dynamical system. There is a parameter \( \lambda \) associated with a dynamical system which is the tightness (marked by the number of turns) of the faucet handle. At first, you observe that the time between drips is constant at 1 second between drops. As you unscrew the faucet handle past the 1/4-turn mark, the drip times begin to exhibit period 2 behavior (e.g., 0.7, 0.9, 0.7, 0.9, seconds, etc.)

   We say that there is a bifurcation at \( \lambda_1 = 0.25 \) turns.
   Then as you unscrew the handle past the 1/2-turn mark, it begins to exhibit period 4 behavior. Hence we’ll say \( \lambda_2 = 0.5 \) turns.

   Although you don’t know the underlying dynamical system, you can still make predictions! Using Feigenbaum’s universal constant, estimate \( \lambda_3 \), the position of the handle at which the next period-doubling will occur.
   Now try this with a real faucet and see if you can observe period-doubling.