Due: Wednesday, October 26

HMC Math 142 Fall 2011
Prof. Gu
Problem Set 6
Start this assignment before Sunday night!

Read:
- Baby Do Carmo, Differential Geometry of Curves and Surfaces: Sections 2-4, 2-5, 2-6 and Section 5-10 on Abstract surfaces (starting on page 425)
- Handouts 8 and 9
- Lecture Notes

Do:

A: Please do a thorough review for your Midterm I.

B: Problems from Lectures
- a) Let $S$ be a subset of $\mathbb{R}^3$. Show that $S$ is regular surface if and only if $S$ is locally diffeomorphic to $\mathbb{R}^2$.
- b) Find five examples of regular surfaces such that each of them can be represented as a surface of revolution. Write down specifically for each example the generating curve, the rotation axis, and the parameterization (as a map) for the surface (including the domain of the map).

C: Other Problems
- a) Problem 10 on page 81, Section 2-3, Baby Do Carmo.
- b) Problem 9 on page 89, Section 2-4, Baby Do Carmo.
- c) Problem 15 on page 90, Section 2-4, Baby Do Carmo.
- d) Problem 18 on page 90, Section 2-4, Baby Do Carmo.
- e) Problem 1 on page 99, Section 2-5, Baby Do Carmo.
• f) Problem 3 on page 99, Section 2-5, Baby Do Carmo.
• g) Problem 9 on page 100, Section 2-5, Baby Do Carmo.

D: Extra Credit Problems

• a) Let \( T \subset \mathbb{R}^3 \) be a torus of revolution with center in \((0,0,0) \in \mathbb{R}^3\) and let \( A(x, y, z) = (-x, -y, -z) \). Let \( K \) be the quotient space of the torus \( T \) by the equivalence relation \( p \sim A(p) \). Can you tell what surface \( K \) is?

• b) Show that \( K \) is a differentiable 2-dimensional manifold.

• c) Show that \( K \) is non orientable in two different ways.